

Worksheet 11
Double Integrals

1. Evaluate the following integrals:

a). $\int_1^3 \int_0^2 x^3 y \, dy \, dx$

b). $\int_0^2 \int_1^3 x^3 y \, dy \, dx$

c). $\int_0^1 \int_0^2 (x + 4y^3) \, dx \, dy$

d). $\int_0^1 \int_2^3 \sqrt{x + 4y} \, dx \, dy$

e). $\int_1^2 \int_0^4 \frac{dy \, dx}{x + y}$

f). $\int_1^2 \int_1^2 \ln(xy) \, dy \, dx.$

2. Let $f(x, y) = mxy^2$, where m is a constant. Given that:

$$\iint_R f(x, y) \, dA = 1,$$

where R is the rectangle $R = [0, 1] \times [0, 2]$, find m .

3. For each of the functions below, compute the integral of $f(x, y)$ over the given region R and sketch the region.

a). $f(x, y) = x^3$; $R: 0 \leq x \leq 2$; $x^2 \leq y \leq 4$.

b). $f(x, y) = x^2 y$; $R: 1 \leq x \leq 3$; $x \leq y \leq 2x + 1$.

c). $f(x, y) = 1$; $R: 0 \leq x \leq 1$; $1 \leq y \leq e^x$.

d). $f(x, y) = 2xy$; $R: 0 \leq y \leq 1$; $y^2 \leq x \leq y$.

e). $f(x, y) = \frac{\sin(y)}{y}$; $R: 1 \leq y \leq 2$; $y \leq x \leq 2y$.

f). $f(x, y) = e^{y^2}$; $R: 0 \leq x \leq 4$; $\frac{x}{2} \leq y \leq 2$.