Math 2551 (L1-L3)
3/14/2016

Worksheet 13 - Substitution in Double Integrals

1. Sketch and describe the region in the (u, v)-plane obtained by applying the transformation

$$u = 2x - y; v = x + y$$

to the triangle with vertices $(1, 2)$, $(2, 1)$, and $(3, 4)$ in the (x, y)-plane.

2. Sketch and describe the region in the (u, v)-plane obtained by applying the transformation

$$u = x/y; v = xy; x, y > 0$$

to the square $[1, 2] \times [1, 2]$ in the (x, y)-plane.

3. Compute

$$\iint_R \cos\left(\frac{y-x}{y+x}\right) dA,$$

where R is the trapezoidal region in the (x, y)-plane with vertices $(1, 0)$, $(2, 0)$, $(0, 2)$, and $(0, 1)$.
Hint: Use the substitution $u = y - x$; $v = y + x$.

4. Compute

$$\iint_R x^2 \, dA,$$

where R is the region in the (x, y)-plane bounded by the ellipse $9x^2 + 4y^2 = 36$, by using the substitution $u = x/2$ and $v = y/3$.

5. Compute

$$\iint_R \left(\frac{x-y}{x+y+2}\right)^2 dA,$$

where R is the square in the (x, y)-plane with vertices $(0, 1)$, $(-1, 0)$, $(0, -1)$, $(1, 0)$.
Hint: Use the substitution $u = x - y$, $v = x + y + 2$.

6. Compute

$$\iint_R e^{\frac{x+y}{4x+y}} \, dA,$$

where R is the region in the (x, y)-plane determined by

$$R = \{(x, y) : \ 1 \leq 4x + y \leq 2; \ x \geq 0; \ y \geq 0\}.$$

Hint: Use the substitution $u = x + y$; $v = 4x + y$.