Worksheet 14

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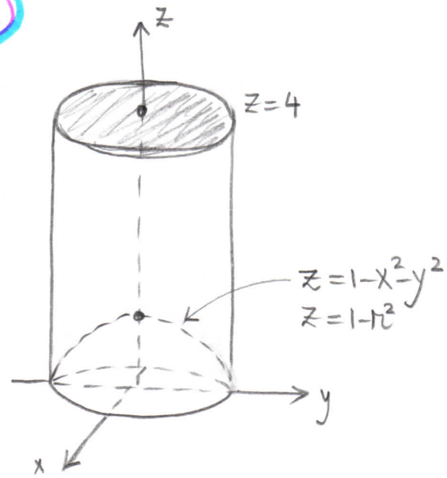


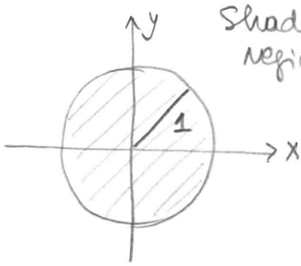
Shadow region: where does $z=1-r^2$ intersect the xy-plane? $(z=0)$

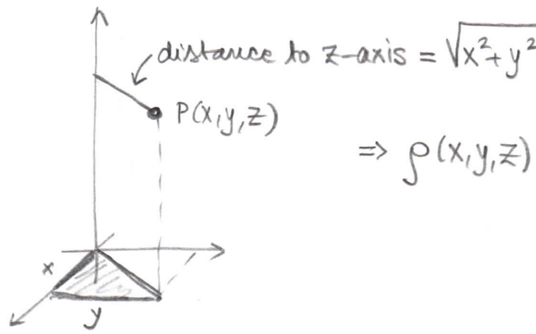
$$1-r^2=0 \Rightarrow \boxed{r=1}$$

Volume integral over D:

$$\int_0^{2\pi}\int_0^1\int_{1-r^2}^4 dz\, r\, dr\, d\theta$$

Shadow region:





$$\Rightarrow \rho(x,y,z)=\sqrt{x^2+y^2}=r$$

$$\Rightarrow m=\iiint_D \rho\, dV = \int_0^{2\pi}\int_0^1\int_{1-r^2}^4 dz\, r^2\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^1 \left(4-(1-r^2)\right)r^2\, dr\, d\theta = 2\pi\int_0^1 (3r^2+r^4)\, dr$$

$$= 2\pi\left(r^3+\frac{r^5}{5}\right)\Big|_0^1 = \frac{12\pi}{5}$$

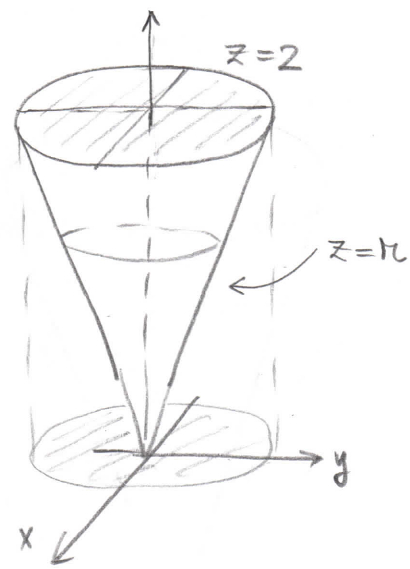
② $\displaystyle\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2}(x^2+y^2)\,dz\,dy\,dx = ?$

<u>Solid</u>: $\sqrt{x^2+y^2} \le z \le 2$

$-\sqrt{4-x^2} \le y \le \sqrt{4-x^2}$

$-2 \le x \le 2$

$z=\sqrt{x^2+y^2}$ Cone



Shadow region of D: $r \le 2$

$$\Rightarrow \text{Integral} = \int_0^{2\pi}\int_0^2\int_r^2 r^2\,dz\,r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^2\int_r^2 r^3\,dz\,dr\,d\theta$$

$$= 2\pi\int_0^2 (2r^3-r^4)\,dr$$

$$= 2\pi\left(\frac{1}{2}r^4-\frac{r^5}{5}\right)\Big|_0^2$$

$$= 2\pi\left(8-\frac{32}{5}\right) = \frac{16\pi}{5}.$$

③ $\displaystyle\iiint_B e^{(x^2+y^2+z^2)^{3/2}}\,dV$; B = unit ball : $B=\{(x,y,z): x^2+y^2+z^2\le 1\}$

<u>Spherical Coordinates</u>: $\displaystyle\int_0^{\pi}\int_0^{2\pi}\int_0^1 e^{(\rho^2)^{3/2}}\rho^2\sin\phi\,d\rho\,d\theta\,d\phi$

$$= \int_0^{\pi}\int_0^{2\pi}\int_0^1 e^{\rho^3}\rho^2\sin\phi\,d\rho\,d\theta\,d\phi$$

$$= 2\pi\left(-\cos\phi\Big|_0^{\pi}\right)\left(\frac{1}{3}e^{\rho^3}\Big|_0^1\right)$$

$$= 2\pi(1+1)\,\frac{1}{3}(e-1) = \boxed{\frac{4\pi}{3}(e-1)}$$