Math 2551 (L1-L3)
4/3/2016

Worksheet 15

Part I - Line Integrals

1. Find $\int_C xy^4\, ds$, where C is the right half of the circle $x^2+y^2=16$.

2. Find $\int_C (2+x^2y)\, ds$, where C is the upper half of the unit circle $x^2+y^2=1$..

3. Find $\int_C \frac{x^2}{y^{4/3}}\, ds$, where C is the curve: $\vec{r}(t)=t^2\vec{i}+t^3\vec{j}$, $-3\le t\le 1$.

4. Find the line integral of $f(x,y,z)=xyz$ along the curve C given by the helix:

$$\vec{r}(t)=\cos(t)\vec{i}+\sin(t)\vec{j}+3t\vec{k},\ 0\le t\le 4\pi.$$

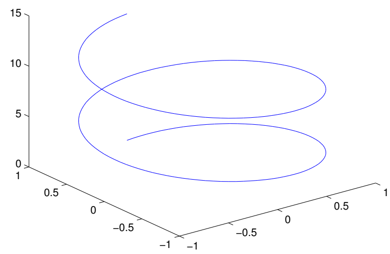

Figure 1: Helix

Part II - Flow, Circulation and Flux

1. Find the work done by the force field $\mathbf{F}(x,y)=x^2\mathbf{i}-xy\mathbf{j}$ to move a particle along the quarter circle $\mathbf{r}(t)=\cos(t)\mathbf{i}+\sin(t)\mathbf{j}$, $0\le t\le \frac{\pi}{2}$.

2. Find the flow of the field $\mathbf{F}(x,y,z)=xy\mathbf{i}+yz\mathbf{j}+zx\mathbf{k}$ along the curve C: $\mathbf{r}(t)=t\mathbf{i}+t^2\mathbf{j}+t^3\mathbf{k}$, $0\le t\le 1$.

3. Find the flow of the field $\mathbf{F}(x,y)=x^2y^3\mathbf{i}-y\sqrt{x}\mathbf{j}$ along the curve C: $\mathbf{r}(t)=t^2\mathbf{i}-t^3\mathbf{j}$, $0\le t\le 1$.

4. Find the flow of the field $\mathbf{F}(x,y,z)=\sin(x)\mathbf{i}+\cos(y)\mathbf{j}+xz\mathbf{k}$ along the curve C: $\mathbf{r}(t)=t^3\mathbf{i}-t^2\mathbf{j}+t\mathbf{k}$, $0\le t\le 1$.

5. Consider the vector field $\mathbf{F}(x,y)=(x-y)\mathbf{i}+(xy)\mathbf{j}$ (see Figure 2).

a). Find the flow of $\mathbf{F}$ along C_1, where C_1 is the arc of the circle $x^2+y^2=4$ traversed counterclockwise from $(2,0)$ to $(0,-2)$.

b). Find the flow of $\mathbf{F}$ along C_2, where C_2 is the arc of the circle $x^2+y^2=4$ traversed counterclockwise from $(0,-2)$ to $(2,0)$.

c). Find the *circulation* of $\mathbf{F}$ around the circle C: $x^2+y^2=4$.

d). Find the *flux* of $\mathbf{F}$ around the circle C: $x^2+y^2=4$.

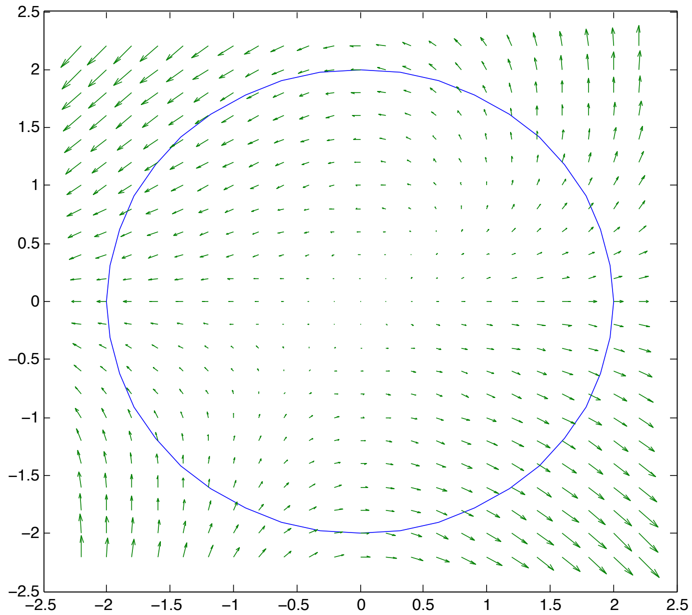

Figure 2: The vector field $\mathbf{F}$ and the curve C in Problem II.5.

6. Find $\int_C \mathbf{F} \cdot d\mathbf{r}$, where

$$\mathbf{F}(x, y) = \left\langle e^{x-1}, xy \right\rangle \quad \text{and} \quad \mathbf{r}(t) = \left\langle t^2, t^3 \right\rangle, \; 0 \leq t \leq 1.$$

7. Find the work done by the force field $\mathbf{F}(x, y) = \langle x, y + 2 \rangle$ to move a particle along an arch of the cycloid (See Figure 3):

$$\text{C}: \; \mathbf{r}(t) = \langle t - \sin(t), 1 - \cos(t) \rangle \, ; \; 0 \leq t \leq 2\pi.$$

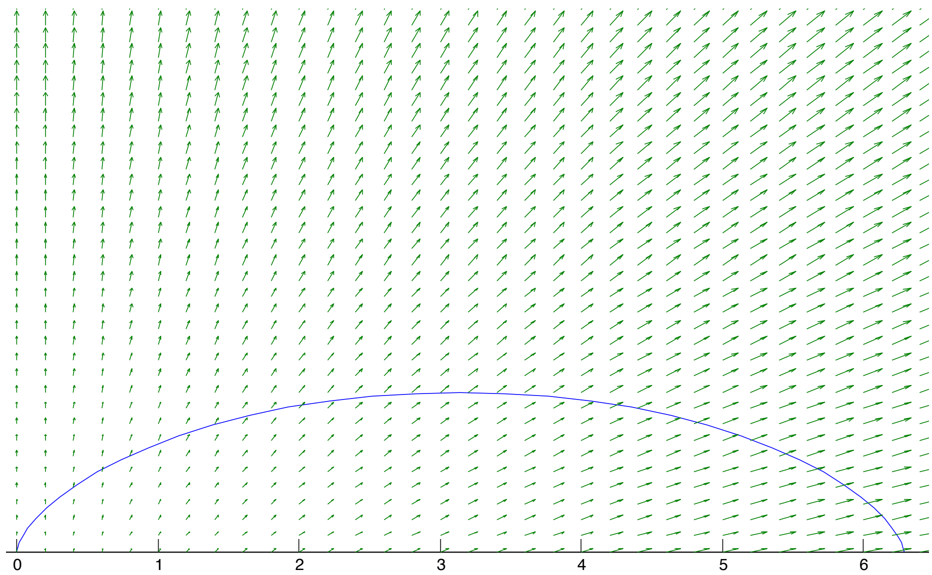

Figure 3: The vector field $\mathbf{F}$ and the curve C in Problem II.7.

8. Figure 4 shows a plot of the so-called "vortex" vector field:

$$\mathbf{F}(x, y) = \left\langle \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right\rangle.$$

Find the circulation and the flux of this field around the circle C, centered at the origin, of radius 2, traversed counterclockwise.

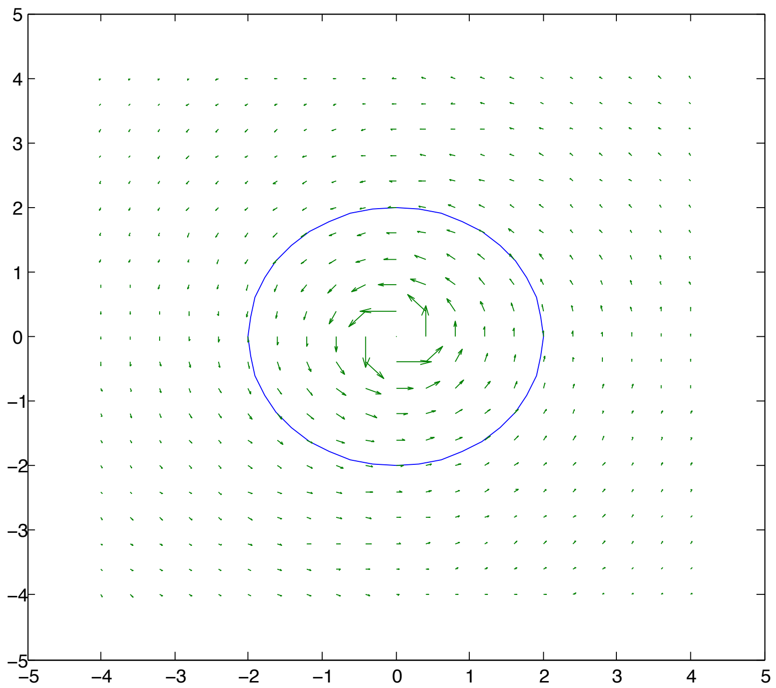

Figure 4: The "vortex" vector field $\mathbf{F}$ and the curve C in Problem II.8.