Math 2551 (L1-L3)
04/11/2016

Worksheet 17 - Green's Theorem

1. Use Green's Theorem to compute:

$$\oint_C x^4\,dx + xy\,dy,$$

where C is the triangle with vertices $(0,0)$, $(1,0)$, and $(0,1)$ - positively oriented.

2. Find:

$$\oint_C (3y - e^{\sin x})\,dx + (7x + \sqrt{y^4+1})\,dy,$$

where C is the circle $x^2 + y^2 = 9$, positively oriented. Give a little thought to computing this using the original definition of line integrals and parametric curves, and compare to the simplicity of using Green's Theorem.

3. Use the area formula we deduced from Green's Theorem:

$$A = \frac{1}{2}\oint_C x\,dy - y\,dx,$$

to compute the area enclosed by an ellipse. Recall that the ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

is parametrized by $x = a\cos t$, $y = b\sin t$, $0 \le t \le 2\pi$.

4. The *lemniscate*, or "figure eight" curve pictured in Figure 1 is given by:

$$x^4 = x^2 - y^2.$$

a). Check that

$$\mathbf{r}(t) = \langle \sin t, \sin t\cos t\rangle\,,\ \ 0 \le t \le \pi$$

is a parametrization of the right lobe of the curve.

b). Use this parametrization and Green's Theorem area formula to compute the area enclosed by the right lobe of the lemniscate.

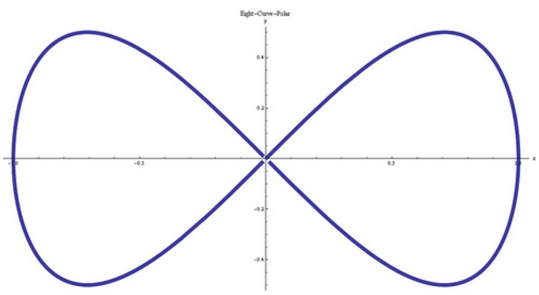

Figure 1: The Lemniscate