## Worksheet 2 - Vectors and the Dot Product

1. Classify the quantities below as either scalar or vector quantities:
a). Velocity of a car.
b). Speed of a car.
c). Number of students at Georgia Tech.
d). Mass of an orange.
e). Weight of an orange.
2. Suppose that $\|\mathbf{u}\|=\sqrt{2}$ and $\|\mathbf{v}\|=1$.
a). Find $\mathbf{u} \cdot \mathbf{v}$, if the angle $\theta$ between $\mathbf{u}$ and $\mathbf{v}$ is $\theta=\pi / 4$.
b). Find $\mathbf{u} \cdot \mathbf{v}$, if the angle $\theta$ between $\mathbf{u}$ and $\mathbf{v}$ is $\theta={ }^{3 \pi} / 4$.
3. Prove that if:

$$
\|\mathbf{u}+\mathbf{v}\|^{2}=\|\mathbf{u}\|^{2}+\|\mathbf{v}\|^{2}
$$

then $\mathbf{u}$ and $\mathbf{v}$ are orthogonal.
4. Let $\mathbf{u}$ and $\mathbf{v}$ be unit vectors, with $\mathbf{u}$ making an angle of $45^{\circ}$ with the positive $x$-axis, and $\mathbf{v}$ making an angle of $60^{\circ}$ with the positive $x$-axis. Use the dot product of these two vectors to compute $\cos \left(15^{\circ}\right)$.
5. The vector $\mathbf{b}=\langle 2,2,1\rangle$ makes an angle of $\pi / 6$ with the vector $\mathbf{a}$. Given that $\mathbf{a} \cdot \mathbf{b}=6$, find $\|\mathbf{a}\|$.
6. Consider the points $P(a, 1,-1), Q(0,1,1)$ and $R(a,-1,3)$, for some real number $a$. Find $a$ such that the angle $\angle P Q R$ is a right angle.
7. Determine whether or not the pairs of vectors are orthogonal:
a). $\mathbf{u}=\langle\cos \theta, \sin \theta,-1\rangle$ and $\mathbf{v}=\langle\cos \theta, \sin \theta, 1\rangle$.
b). $\mathbf{u}=\langle\cos \theta, \sin \theta,-1\rangle$ and $\mathbf{v}=\langle\sin \theta,-\cos \theta, 1\rangle$.
8. Prove the Cauchy-Schwartz inequality:

$$
|\mathbf{u} \cdot \mathbf{v}| \leq\|\mathbf{u}\|\|\mathbf{v}\| .
$$

9. Prove the Parallelogram Law:

$$
\|\mathbf{u}+\mathbf{v}\|^{2}+\|\mathbf{u}-\mathbf{v}\|^{2}=2\left(\|\mathbf{u}\|^{2}+\|\mathbf{v}\|^{2}\right)
$$

