

Worksheet 4 - Intersecting Planes

- The angle between two planes in space is defined to be the angle between their normal vectors.
- Two planes are parallel if and only if their normal vectors are parallel.
- Two non-parallel planes intersect in a line.

Example: Consider the planes:

$$3x - 6y - 2z = 15, \quad \vec{n}_1 = \langle 3, -6, -2 \rangle$$

and:

$$2x + y - 2z = 5. \quad \vec{n}_2 = \langle 2, 1, -2 \rangle$$

- a). Find the angle between the planes.
 b). Are the two planes parallel? If not, find parametric equations for the line of intersection of the two planes.

$$a). \cos \theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| |\vec{n}_2|} = \frac{6 - 6 + 4}{\sqrt{49} \sqrt{9}} = \frac{4}{21} \Rightarrow \theta = \cos^{-1}\left(\frac{4}{21}\right)$$

b). The planes are not parallel, because their normal vectors are not scalar multiples of each other.

Line of Intersection L:

→ Parallel Vector: The line L is perpendicular to both \vec{n}_1 & \vec{n}_2 , so a vector parallel to the line is $\vec{v} = \vec{n}_1 \times \vec{n}_2$:

$$\vec{v} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -6 & -2 \\ 2 & 1 & -2 \end{vmatrix} = \langle 14, 2, 15 \rangle$$

→ Point on L: Just find any point that belongs to both planes.

For example, take $z = 0$ and solve for x & y :

$$\begin{cases} 3x - 6y = 15 \\ 2x + y = 5 \end{cases} \quad \begin{cases} x - 2y = 5 \\ 2x + y = 5 \end{cases} \quad \begin{array}{l} x - 2y = 5 \\ \times 2 \quad 4x + 2y = 10 \\ \hline \oplus \quad 5x = 15 \end{array} \Rightarrow x = 3 \Rightarrow y = -1$$

So a point on L is: $(3, -1, 0)$

Parametric Equations:

$$\begin{cases} x = 3 + 14t \\ y = -1 + 2t \\ z = 15t \end{cases}$$