Worksheet 5 – Solutions

① $\vec{r}(t) = \langle t\sin t + \cos t\, ,\ -t\cos t + \sin t \rangle$; $-\sqrt{2} \leq t \leq 2$

a). $\vec{v}(t) = \langle \cancel{\sin t} + t\cos t - \cancel{\sin t}\, ,\ -\cancel{\cos t} + t\sin t + \cancel{\cos t} \rangle$
$= \langle t\cos t\, ,\ t\sin t \rangle$

b). $|\vec{v}(t)| = \sqrt{t^2\cos^2 t + t^2\sin^2 t} = \sqrt{t^2} = |t|$

$\vec{T}(t) = \dfrac{\vec{v}(t)}{|\vec{v}(t)|} = \left\langle \dfrac{t\cos t}{|t|}\, ,\ \dfrac{t\sin t}{|t|} \right\rangle$

! Be careful with $\sqrt{t^2}$!

$\boxed{\sqrt{t^2} = |t|}$ ← correct

$\sqrt{t^2} = t$ ← incorrect!

Example : $\sqrt{9} = \sqrt{(-3)^2} \neq -3$
$= 3 = |-3|$

② $\vec{r}(t) = \langle 3\cos t\, ,\ 3\sin t\, ,\ t \rangle$

a). $\vec{v}(t) = \langle -3\sin t\, ,\ 3\cos t\, ,\ 1 \rangle$

b). $|\vec{v}(t)| = \sqrt{9\sin^2 t + 9\cos^2 t + 1} = \sqrt{10}$

$\vec{T}(t) = \dfrac{1}{\sqrt{10}} \langle -3\sin t, 3\cos t, 1 \rangle$

③ $\dfrac{d\vec{r}}{dt} = \left\langle 6\sqrt{t+1}\, ,\ 4e^{-t},\ \dfrac{1}{t+1} \right\rangle$

$\vec{r}(t) = \left\langle 6 \cdot \tfrac{2}{3}(t+1)^{3/2} + C_1\, ;\ -4e^{-t} + C_2\, ,\ \ln(t+1) + C_3 \right\rangle$

$$\left.\begin{aligned} \vec{r}(0) &= \langle 4 + C_1\, ,\ -4 + C_2\, ,\ C_3 \rangle \\ \vec{r}(0) &= \langle 0\, ,\ 0\, ,\ 1 \rangle \end{aligned}\right\} \Rightarrow C_1 = -4\ ;\ C_2 = 4;\ C_3 = 1$$

$$\boxed{\vec{r}(t) = \langle 4(t+1)^{3/2} - 4\, ,\ -4e^{-t} + 4\, ,\ \ln(t+1) + 1 \rangle}$$

④ $\vec{r}(t) = \left(\ln \frac{t}{6}\right)\vec{i} + \left(\frac{t-6}{t+7}\right)\vec{j} + \left(t \ln \frac{t}{6}\right)\vec{k}$; $t=6$

$$\vec{v}(t) = \left\langle \frac{1}{t/6} \cdot \frac{1}{6},\ \frac{(t+7)-(t-6)}{(t+7)^2},\ \ln\frac{t}{6} + t \cdot \frac{1}{t/6}\frac{1}{6} \right\rangle$$

$$= \left\langle \frac{1}{t},\ \frac{13}{(t+7)^2},\ \ln\left(\frac{t}{6}\right) + 1 \right\rangle$$

$\vec{v}(6) = \left\langle \frac{1}{6}, \frac{1}{13}, 1 \right\rangle$ (vector parallel to the line)

Point: $\vec{r}(6) = \langle 0, 0, 0 \rangle \Rightarrow (0,0,0)$ (point on the line)

Equations for the line:

$$\boxed{\begin{cases} x = \frac{1}{6}\tau \\ y = \frac{1}{13}\tau \\ z = \tau \end{cases}}$$

(5) $\vec{r}(t) = \langle \cos t \,,\, \ln(4-t),\, \sqrt{t+1} \rangle$

Domain of $\underline{\cos t}$: $\mathbb{R}$

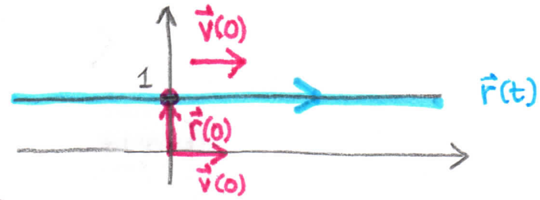
Domain of $\underline{\ln(4-t)}$: $4-t>0$ $\quad (-\infty, 4)$

Domain of $\underline{\sqrt{t+1}}$: $t+1 \geq 0 \quad t \geq -1 \quad [-1, \infty)$

$\Big\}$ Domain of $\vec{r}(t)$: $[-1, 4)$

(6) a). $\vec{r}(t) = \langle t, 1 \rangle$

Parametric equations: $\begin{cases} x = t \\ y = 1 \end{cases}$



Orientation? Plot a velocity vector:

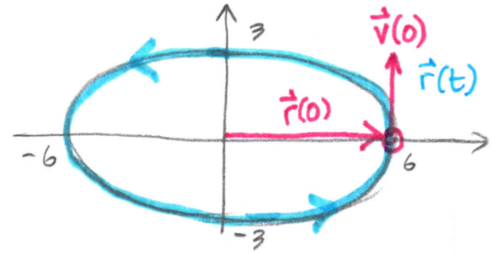
$t=0 \Rightarrow \vec{r}(0) = \langle 0, 1 \rangle$

$\vec{v}(t) = \langle 1, 0 \rangle \Rightarrow \vec{v}(0) = \langle 1, 0 \rangle$

b). $\vec{r}(t) = \langle 6\cos t,\, 3\sin t \rangle$

Parametric Equations: $\begin{cases} x = 6\cos t \\ y = 3\sin t \end{cases}$

$\left.\begin{aligned} \Rightarrow \cos t &= \frac{x}{6} \\ \sin t &= \frac{y}{3} \end{aligned}\right\} \Rightarrow \boxed{\left(\frac{x}{6}\right)^2 + \left(\frac{y}{3}\right)^2 = 1}$ $\underline{\underline{\text{Ellipse}}}$

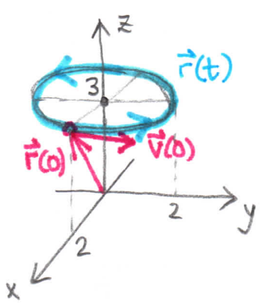


$\vec{v}(t) = \langle -6\sin t,\, 3\cos t \rangle$

$\vec{r}(0) = \langle 6, 0 \rangle$

$\vec{v}(0) = \langle 0, 3 \rangle$

c). $\vec{r}(t) = \langle 2\cos t,\, 2\sin t,\, 3 \rangle$



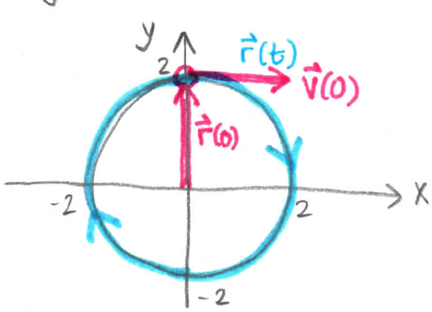
$\vec{v}(t) = \langle -2\sin t,\, 2\cos t,\, 0 \rangle$

$\vec{r}(0) = \langle 2, 0, 3 \rangle$

$\vec{v}(0) = \langle 0, 2, 0 \rangle$

d). $\vec{r}(t) = \langle 2\sin t,\, 2\cos t \rangle$

$\begin{cases} x = 2\sin t \\ y = 2\cos t \end{cases} \Rightarrow x^2 + y^2 = 4$



$\vec{v}(t) = \langle 2\cos t,\, -2\sin t \rangle$

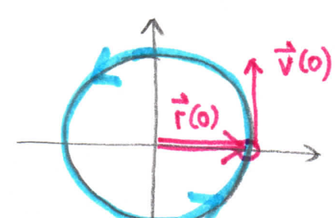
$\vec{r}(0) = \langle 0, 2 \rangle$

$\vec{v}(0) = \langle 2, 0 \rangle$

Circle of radius 2, oriented $\underline{\underline{\text{clockwise}}}$

Compare to:

$\vec{r}(t) = \langle 2\cos t,\, 2\sin t \rangle$



$\vec{r}(0) = \langle 2, 0 \rangle$

$\vec{v}(t) = \langle -2\sin t,\, 2\cos t \rangle$

$\vec{v}(0) = \langle 0, 2 \rangle$

Circle of radius 2 oriented $\underline{\underline{\text{counterclockwise}}}$