Worksheet 5

1. Consider the curve:

$$\vec{r}(t) = (t\sin t + \cos t)\,\vec{i} + (-t\cos t + \sin t)\,\vec{j}; \quad -\sqrt{2} \le t \le 2.$$

a). Find the velocity $\vec{v}(t)$.

b). Find the unit tangent vector $\vec{T}(t) = \frac{\vec{v}(t)}{|\vec{v}(t)|}$.

2. Consider the curve:

$$\vec{r}(t) = 3\cos t\,\vec{i} + 3\sin t\,\vec{j} + t\vec{k}.$$

a). Find the velocity $\vec{v}(t)$.

b). Find the unit tangent vector $\vec{T}(t)$.

3. Given that:

$$\frac{d\vec{r}}{dt} = 6\sqrt{t+1}\,\vec{i} + 4e^{-t}\,\vec{j} + \frac{1}{t+1}\,\vec{k},$$

$$\vec{r}(0) = \vec{k},$$

find $\vec{r}(t)$.

4. Find parametric equations for the line tangent to the curve:

$$\vec{r}(t) = \left(\ln\frac{t}{6}\right)\vec{i} + \left(\frac{t-6}{t+7}\right)\vec{j} + \left(t\ln\frac{t}{6}\right)\vec{k}$$

at the value $t = 6$ of the parameter.

5. Find the domain of the vector-valued function:

$$\vec{r}(t) = \left\langle \cos t, \ln(4-t), \sqrt{t+1} \right\rangle$$

6. Sketch the following curves, with their orientations:

   a). $\vec{r}(t) = \langle t, 1 \rangle$

   b). $\vec{r}(t) = \langle 6\cos t, 3\sin t \rangle$

   c). $\vec{r}(t) = \langle 2\cos t, 2\sin t, 3 \rangle$

   d). $\vec{r}(t) = \langle 2\sin t, 2\cos t \rangle$