Worksheet 6

1. Consider the curve:

$$\mathbf{r}(t) = \left\langle t^2, \sin t, t \right\rangle ; \;\; 3 \leq t \leq 9.$$

Give a new parametrization $\mathbf{r}_1(s)$ of this curve in terms of the parameter s, given by $t = e^s$.

2. Find the *arc length parametrization* for the line $\mathbf{r}(t) = \langle 2t, 1-2t, t \rangle$, taking $(0,1,0)$ as your basepoint.

3. Recall the definition of *curvature* of a curve $\mathbf{r}(t)$:

$$\kappa(t) = \left| \frac{d\mathbf{T}}{ds} \right| = \frac{1}{|\mathbf{v}(t)|} \left| \frac{d\mathbf{T}}{dt} \right| ,$$

where $\mathbf{v}(t)$ is the velocity and $\mathbf{T}(t)$ is the unit tangent vector.

a). Show that curvature can also be expressed as:

$$\kappa(t) = \frac{|\mathbf{v}(t) \times \mathbf{a}(t)|}{|\mathbf{v}(t)|^3},$$

where $\mathbf{a}(t)$ is the acceleration. As highlighted in the next example, this latter formula is often easier to use than the original definition. Follow the steps below to prove this new formula:

- Show that:

$$\mathbf{v}(t) = \frac{ds}{dt}\mathbf{T}(t) \quad \text{and} \quad \mathbf{a}(t) = \frac{d^2 s}{dt^2}\mathbf{T}(t) + \frac{ds}{dt}\frac{d\mathbf{T}}{dt}.$$

- Use the expressions above to show that

$$\mathbf{v}(t) \times \mathbf{a}(t) = |\mathbf{v}(t)|^2 \left(\mathbf{T}(t) \times \frac{d\mathbf{T}}{dt} \right).$$

- Take the length of both sides above to show that

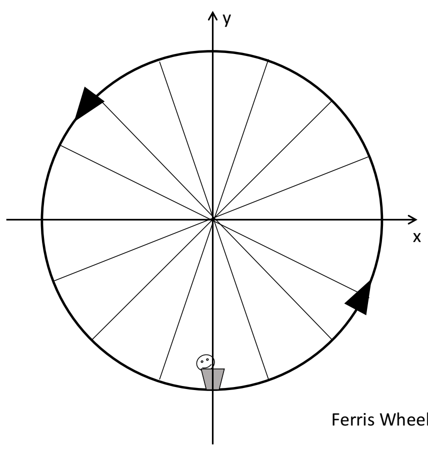
$$|\mathbf{v}(t) \times \mathbf{a}(t)| = |\mathbf{v}(t)|^2 \left| \frac{d\mathbf{T}}{dt} \right|.$$

- Conclude the new formula for curvature.

b). Compute the curvature of $\mathbf{r}(t) = \left\langle t, t^2, t^3 \right\rangle$.

4. At what point on the curve $\mathbf{r}(t) = \left\langle t^3, 3t, t^4 \right\rangle$ is the normal plane parallel to the plane $6x+6y-8z = 1$? Recall that the normal plane is the one determined by $\mathbf{B}$ and $\mathbf{N}$.

5. Suppose a Ferris wheel is a circle of radius $R = 30m$. At time $t = t_0$, a person is seated in the car at the lowest point of the wheel (see figure), which is rotating counterclockwise with a speed of $30m/min$ and is slowing down at a rate of $15m/min^2$ at this time. Find the acceleration vector $\mathbf{a}(t_0)$. How would your answer change if the wheel was rotating *clockwise*?



Ferris Wheel