1) a) $f(x,y) = xy^3 + x^2y^2$

$\frac{\partial f}{\partial x} = y^3 + 2xy^2$; $\frac{\partial f}{\partial y} = 3xy^2 + 2x^2y$

b) $f(x,y) = xe^{2x+3y}$

$\frac{\partial f}{\partial x} = e^{2x+3y} + 2xe^{2x+3y}$; $\frac{\partial f}{\partial y} = 3xe^{2x+3y}$

c) $f(x,y) = \frac{x-y}{x+y}$

$\frac{\partial f}{\partial x} = \frac{1(x+y) - 1(x-y)}{(x+y)^2} = \frac{2y}{(x+y)^2}$; $\frac{\partial f}{\partial y} = \frac{-1(x+y) - 1(x-y)}{(x+y)^2} = \frac{-2x}{(x+y)^2}$

d) $f(x,y) = 2x\sin(x^2y)$

$\frac{\partial f}{\partial x} = 2\sin(x^2y) + 4x^2y\cos(x^2y)$; $\frac{\partial f}{\partial y} = 2x^3\cos(x^2y)$

e) $f(x,y,z) = x\cos z + x^2y^3e^z$

$\frac{\partial f}{\partial x} = \cos z + 2xy^3e^z$; $\frac{\partial f}{\partial y} = 3x^2y^2e^z$; $\frac{\partial f}{\partial z} = x^2y^3e^z$

2) $u(x,y) = \ln(1+xy^2)$

$\frac{\partial u}{\partial x} = \frac{y^2}{1+xy^2}$; $\frac{\partial^2 u}{\partial x^2} = -\frac{y^2}{(1+xy^2)^2} \cdot y^2 = -\frac{y^4}{(1+xy^2)^2}$

$\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) = \frac{2y(1+xy^2) - y^2(2xy)}{(1+xy^2)^2} = \frac{2y + 2xy^3 - 2xy^3}{(1+xy^2)^2} = \frac{2y}{(1+xy^2)^2}$

$\Rightarrow 2\frac{\partial^2 u}{\partial x^2} + y^3\frac{\partial^2 u}{\partial y \partial x} = -\frac{2y^4}{(1+xy^2)^2} + \frac{2y^4}{(1+xy^2)^2} = \underline{\underline{0}}$

(3) $g(s,t) = f(s^2-t^2, t^2-s^2)$

Let $x = s^2-t^2$ and $y = t^2-s^2$. Then $g(s,t) = f(x,y)$, and by the Chain Rule:

$$\frac{\partial g}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s} = \left(\frac{\partial f}{\partial x}\right)(2s) + \left(\frac{\partial f}{\partial y}\right)(-2s)$$

$$\frac{\partial g}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t} = \left(\frac{\partial f}{\partial x}\right)(-2t) + \left(\frac{\partial f}{\partial y}\right)(2t)$$

$$\Rightarrow t\frac{\partial g}{\partial s} + s\frac{\partial g}{\partial t} = \cancel{\left(\frac{\partial f}{\partial x}\right)(2st)} + \cancel{\left(\frac{\partial f}{\partial y}\right)(-2st)} + \cancel{\left(\frac{\partial f}{\partial x}\right)(-2st)} + \cancel{\left(\frac{\partial f}{\partial y}\right)(2st)} = 0$$