1.

$f(x,y) = x^2 + \sin(xy)$; find $\vec{u}$ s.t. $(D_{\vec{u}}f)_{(1,0)} = 1$

$\nabla f(x,y) = \left\langle 2x + y\cos(xy),\ x\cos(xy) \right\rangle$

$\nabla f(1,0) = \langle 2, 1 \rangle$

$\vec{u} = \langle a, b \rangle$ unit vector

$$(D_{\vec{u}}f)_{(1,0)} = \nabla f(1,0) \cdot \vec{u} = \langle 2,1\rangle \cdot \langle a,b\rangle = 2a + b$$

$$\begin{cases} 2a+b=1 & \Rightarrow b = 1-2a \\ a^2+b^2=1 & \Rightarrow a^2 + (1-2a)^2 = 1 \end{cases}$$

$$a^2 + \cancel{1} - 4a + 4a^2 = \cancel{1}$$

$$5a^2 - 4a = 0$$

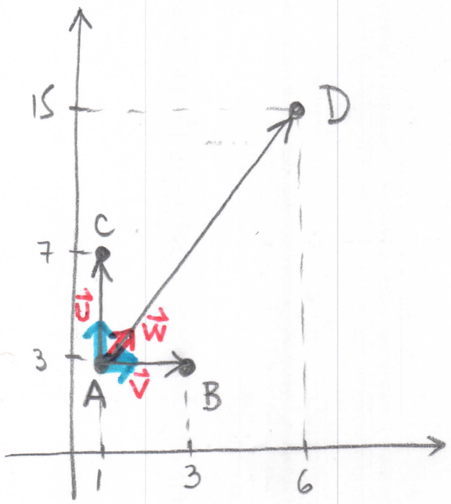
$$a(5a-4) = 0 \Rightarrow a = 0 \text{ or } a = 4/5$$

$a = 0 \Rightarrow b = 1$

$a = \frac{4}{5} \Rightarrow b = 1 - \frac{8}{5} = -\frac{3}{5}$

$$\boxed{\vec{u} = \langle 0, 1 \rangle \text{ or } \vec{u} = \langle 4/5, -3/5 \rangle}$$

2. $f(x,y)$; $A(1,3)$, $B(3,3)$, $C(1,7)$, $D(6,15)$.



Let $\vec{u}$, $\vec{v}$, $\vec{w}$ be the directions of the vectors $\vec{AC}$, $\vec{AB}$, $\vec{AD}$, respectively.

The problem gives us that

$$(D_{\vec{u}}f)_A = 3$$

$$(D_{\vec{v}}f)_A = 26$$

and asks us to find $(D_{\vec{w}}f)_A = ?$

$$\left.\begin{array}{l}\vec{AB} = \langle 2,0\rangle \\ \|\vec{AB}\| = 2\end{array}\right\} \Rightarrow \vec{u} = \langle 1,0\rangle = \vec{i} \Rightarrow (D_{\vec{u}}f)_A = (D_{\vec{i}}f)_A = 3$$

and the directional derivative in the direction of $\vec{i}$ is actually just the partial derivative wrt. x, so this really means: $\boxed{\left(\frac{\partial f}{\partial x}\right)_A = 3}$

$$\left.\begin{array}{l}\vec{AC} = \langle 0,4\rangle \\ \|\vec{AC}\| = 4\end{array}\right\} \Rightarrow \vec{v} = \langle 0,1\rangle = \vec{j} \Rightarrow (D_{\vec{v}}f)_A = (D_{\vec{j}}f)_A = 26$$

Since the directional derivative in the $\vec{j}$ direction is just the partial derivative wrt y, this really means: $\boxed{\left(\frac{\partial f}{\partial y}\right)_A = 26}$

$$\Rightarrow (\nabla f)_A = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right\rangle_A = \langle 3, 26\rangle.$$

$$\left.\begin{array}{l}\vec{AD} = \langle 5,12\rangle \\ \|\vec{AD}\| = 13\end{array}\right\} \Rightarrow \vec{w} = \left\langle \frac{5}{13}, \frac{12}{13}\right\rangle$$

$$\Rightarrow (D_{\vec{w}}f)_A = (\nabla f)_A \cdot \vec{w} = \langle 3, 26\rangle \cdot \left\langle \frac{5}{13}, \frac{12}{13}\right\rangle$$

$$= \frac{15}{13} + 24 = \boxed{\frac{327}{13}}$$