Daily Laplace - 3/30

① $\mathcal{L}\{e^{4t}\cos(3t)\} = ?$ w/ frequency domain

$$= \mathcal{L}\{\cos(3t)\}\Big|_{s\to s-4} = \frac{s}{s^2+9}\Big|_{s\to s-4}$$

$(s>0) \mapsto (s-4>0)$

$$= \frac{s-4}{(s-4)^2+9} \quad ; s>4$$

② $\mathcal{L}^{-1}\left\{\frac{9}{s^2+2}\right\} = 9\cdot\frac{1}{\sqrt{2}}\sin(\sqrt{2}t)$

③ $\mathcal{L}^{-1}\left\{\frac{s}{(s+1)^2-2}\right\}$

$$= \mathcal{L}^{-1}\left\{\frac{(s+1)-1}{(s+1)^2-2}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2-2}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2-2}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{s}{s^2-2}\Big|_{s\to s+1}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s^2-2}\Big|_{s\to s+1}\right\}$$

$$= e^{-t}\mathcal{L}^{-1}\left\{\frac{s}{s^2-2}\right\} - e^{-t}\mathcal{L}^{-1}\left\{\frac{1}{s^2-2}\right\}$$

$$= e^{-t}\cdot\cosh(\sqrt{2}t) - e^{-t}\frac{1}{\sqrt{2}}\sinh(\sqrt{2}t).$$