Daily Laplace- 4/2

$y'' + y = g(t)$; $y(0) = 1$; $y'(0) = 2$

Solution is of the form $y(t) = y_0(t) + (f * g)(t)$. Find y_0 and f.

$$s^2 Y(s) - s\underbrace{y(0)}_{1} - \underbrace{y'(0)}_{2} + Y(s) = G(s)$$

$$(s^2+1)Y(s) = G(s) + s + 2$$

$$Y(s) = \frac{G(s)}{s^2+1} + \frac{s}{s^2+1} + \frac{2}{s^2+1}$$

$$y(t) = \mathcal{L}^{-1}\left\{\frac{G(s)}{s^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} + 2\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}$$

$$= (f*g)(t) + \cos(t) + 2\sin(t)$$

where $f(t) = \sin(t)$

$$\Rightarrow \boxed{\begin{array}{l} y_0(t) = \cos(t) + 2\sin(t) \\ f(t) = \sin(t) \end{array}}$$