

Daily Laplace - 4/4

$$y(t) = t^2 + \int_0^t (t-z)^3 y(z) dz$$

Solve for $y(t)$

$$\downarrow$$
$$= t^3 * y(t)$$

$$y(t) = t^2 + t^3 * y(t)$$

$$Y(s) = \frac{2}{s^3} + \frac{6}{s^4} Y(s)$$

$$\left(1 - \frac{6}{s^4}\right) Y(s) = \frac{2}{s^3}$$

$$\frac{s^4 - 6}{s^4} Y(s) = \frac{2}{s^3} \quad | \cdot \frac{s^4}{s^4 - 6}$$

$$Y(s) = \frac{2s}{s^4 - 6} = \frac{2s}{(s^2 - \sqrt{6})(s^2 + \sqrt{6})}$$
$$= \frac{2s[(s^2 + \sqrt{6}) - (s^2 - \sqrt{6})]}{(s^2 - \sqrt{6})(s^2 + \sqrt{6})} \quad \frac{1}{2\sqrt{6}}$$

$$= \frac{1}{\sqrt{6}} \left(\frac{s}{s^2 - \sqrt{6}} - \frac{s}{s^2 + \sqrt{6}} \right)$$

$$\Rightarrow y(t) = \frac{1}{\sqrt{6}} \left(\cosh(\sqrt{6} t) - \cos(\sqrt{6} t) \right)$$