Daily ODE: 02/12/2022

Solve for $y(x)$:
(Implicit solution OK)

$$[y^2+2x\cos(y)]dx+[2xy-x^2\sin(y)-2y]dy=0$$

Solution: Exact? $M=y^2+2x\cos(y)$ $\quad M_y=2y-2x\sin(y)$

Yes $\quad N=2xy-x^2\sin(y)-2y$ $\quad N_x=2y-2x\sin(y)$

Find Potential: $\begin{cases} \dfrac{\partial f}{\partial x}=y^2+2x\cos(y) \\ \dfrac{\partial f}{\partial y}=2xy-x^2\sin(y)-2y \end{cases}$

Start with x? $\quad f(x,y)=\int(y^2+2x\cos(y))dx$

OK $\quad = y^2x+x^2\cos(y)+g(y)$

$$\left.\begin{aligned} \Rightarrow \frac{\partial f}{\partial y} &= 2yx-x^2\sin(y)+g'(y) \\ N &= 2xy-x^2\sin(y)-2y \end{aligned}\right\} \Rightarrow$$

$\Rightarrow g'(y)=-2y \Rightarrow$ take $g(y)=-y^2$

$f(x,y)=xy^2+x^2\cos(y)-y^2$ $\quad$ Potential

Solution: $\boxed{xy^2+x^2\cos(y)-y^2=C}$

Check: $\frac{d}{dx}\left[\ xy^2+x^2\cos(y)-y^2=C\ \right]$

$$y^2+2xy\cdot y'+2x\cos(y)-x^2\sin(y)\cdot y'-2y\cdot y'=0$$

$$\left[y^2+2x\cos(y)\right]+\left[2xy-x^2\sin(y)-2y\right]\underset{\underset{\frac{dy}{dx}}{\shortparallel}}{y'}=0$$

$$\left[y^2+2x\cos(y)\right]dx+\left[2xy-x^2\sin(y)-2y\right]dy=0$$

Yes (original equation).