Daily ODE - 02/15/2022

Given that $y_1(x) = x$ is a solution to $x^2y'' - 3xy' + 3y = 0$ on $x \in (0, \infty)$, use reduction of order to find a second solution $y_2(x)$, that is linearly independent from $y_1(x)$. Write the general solution to the ODE.

Solution: Look for $u(x)$ such that $y_2(x) = u(x) \cdot x$ is a solution.

$$y_2 = u(x) \cdot x \Rightarrow y_2' = u' \cdot x + u$$

$$y_2'' = u'' \cdot x + u' + u' = u'' \cdot x + 2u'$$

$\Rightarrow x^2 y_2'' - 3x y_2' + 3y_2 = 0$ would become:

$$u'' \cdot x^3 + 2u' \cdot x^2 - 3x^2 u' - \cancel{3xu} + \cancel{3xu} = 0$$

$$x^3 \cdot u'' - x^2 u' = 0 \Rightarrow x u'' - u' = 0$$

Substitution: $w = u'$

$xw' - w = 0$ Separable!

$$x \frac{dw}{dx} = w$$

$$\frac{1}{w} dw = \frac{1}{x} dx$$

$$\ln w = \ln|x| + C \Rightarrow w = C \cdot x$$

$$= \ln(x) + C$$

Take $w = x \Rightarrow u' = x \Rightarrow u = \frac{x^2}{2} \Rightarrow$ Take $\boxed{y_2 = x^3}$ (we only need one).

$(y_2 = u \cdot x)$ (linearly independent from $y_1(x) = x$ on $(0, \infty)$)

Fundamental Set: $\{x, x^3\}$

General Solution: $\boxed{y = c_1 x + c_2 x^3}$