Daily ODE: 02/16/2022

What is the largest possible interval in which a solution to the IVP:

$$(x^2-4)y'' + y' - \sin(x)\,y = 0$$
$$y(\pi) = 1;\ y'(\pi) = 2$$

is guaranteed to exist & be unique?

Solution:

The Existence & Uniqueness Theorem applies as long as all coefficient functions (x^2-4, 1, $-\sin(x)$, 0) are continuous on I (these are all continuous on $\mathbb{R}$) and the leading coefficient $x^2-4 \neq 0$ on I.

$$x^2 - 4 = 0$$
$$(x-2)(x+2) = 0$$
$$x = \pm 2$$

The initial condition is given at $x_0 = \pi > 2$, so the interval is

$$\boxed{(2, \infty)}$$