Laplace transforms of some basic functions

$\mathscr{L}\{1\} = \frac{1}{s};\ s > 0$

$\mathscr{L}\{t^n\} = \frac{n!}{s^{n+1}};\ s > 0$

$\mathscr{L}\{e^{at}\} = \frac{1}{s-a};\ s > a$

$\mathscr{L}\{\sin(at)\} = \frac{a}{s^2+a^2};\ s > 0$

$\mathscr{L}\{\cos(at)\} = \frac{s}{s^2+a^2};\ s > 0$

$\mathscr{L}\{\sinh(at)\} = \frac{a}{s^2-a^2};\ s > |a|$

$\mathscr{L}\{\cosh(at)\} = \frac{s}{s^2-a^2};\ s > |a|$

$\mathscr{L}\{u_a(t)\} = \frac{e^{-as}}{s};\ s > 0$

$\mathscr{L}\{\delta(t-t_0)\} = e^{-st_0}$

$\mathscr{L}\{\delta(t)\} = 1$

Inverse Laplace transforms of some basic functions

$\mathscr{L}^{-1}\left\{\frac{1}{s}\right\} = 1$

$\mathscr{L}^{-1}\left\{\frac{1}{s^n}\right\} = \frac{1}{(n-1)!}t^{n-1}$

$\mathscr{L}^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$

$\mathscr{L}^{-1}\left\{\frac{1}{s^2+a^2}\right\} = \frac{1}{a}\sin(at)$

$\mathscr{L}^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos(at)$

$\mathscr{L}^{-1}\left\{\frac{1}{s^2-a^2}\right\} = \frac{1}{a}\sinh(at)$

$\mathscr{L}^{-1}\left\{\frac{s}{s^2-a^2}\right\} = \cosh(at)$

$\mathscr{L}^{-1}\left\{\frac{e^{-as}}{s}\right\} = u_a(t)$

$\mathscr{L}^{-1}\left\{e^{-st_0}\right\} = \delta(t-t_0)$

$\mathscr{L}^{-1}\left\{1\right\} = \delta(t)$

Properties of the Laplace and Inverse Laplace transform

Translation Theorem I:

$\mathscr{L}\left\{e^{at}f(t)\right\} = F(s-a) = \mathscr{L}\{f(t)\}|_{s\to s-a}$

$\mathscr{L}^{-1}\{F(s-a)\} = \mathscr{L}^{-1}\left\{F(s)|_{s\to s-a}\right\} = e^{at}\mathscr{L}^{-1}\{F(s)\} = e^{at}f(t)$

Translation Theorem II:

$\mathscr{L}\left\{f(t-a)u_a(t)\right\} = e^{-as}F(s) = e^{-as}\mathscr{L}\{f(t)\}$

$\mathscr{L}^{-1}\{e^{-as}F(s)\} = f(t-a)u_a(t) = \mathscr{L}^{-1}\{F(s)\}\big|_{t\to t-a}\, u_a(t)$

Derivatives of Laplace Transforms:

$\mathscr{L}\left\{t^n f(t)\right\} = (-1)^n \frac{d^n}{ds^n}F(s)$

$\mathscr{L}^{-1}\left\{F^{(n)}(s)\right\} = (-1)^n t^n f(t)$

Laplace Transform of Periodic Functions:
If $f(t)$ is piecewise continuous on $[0,\infty)$, of exponential order, and periodic with period T:

$$\mathscr{L}\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt$$

Laplace Transforms of Derivatives:

$$\begin{aligned}
\mathscr{L}\{y'\} &= sY(s) - y(0)\\
\mathscr{L}\{y''\} &= s^2Y(s) - sy(0) - y'(0)\\
&\ \vdots\\
\mathscr{L}\{y^{(n)}(t)\} &= s^nY(s) - s^{n-1}y(0) - s^{n-2}y'(0) - \ldots - sy^{(n-2)}(0) - y^{(n-1)}(0)
\end{aligned}$$

Convolution Theorem:

$\mathscr{L}\{f(t)\star g(t)\} = F(s)G(s) = \mathscr{L}\{f(t)\}\mathscr{L}\{g(t)\}$

$\mathscr{L}^{-1}\{F(s)G(s)\} = f(t)\star g(t)$

$(f\star g)(t) := \int_0^t f(t-\tau)g(\tau)\,d\tau$

Dirac Delta Function:

$\int_0^\infty f(t)\delta(t-t_0)\,dt = f(t_0)$

$(f\star\delta)(t) = f(t)$