Some extra issues from homework problems:

(I) Unlike homogeneous 2^{nd} order IVPs, (Solution exists & is unique!) BVPs can behave in strange ways, such as having no solutions or ∞-many solutions.

Ex: (HW 6, #7 in Part 1)

$$y'' + 4y = 0$$
$$y(0) = 0; \; y(\pi) = 0 \qquad \text{BVP}$$

Char. Eqn.: $m^2 + 4 = 0$

$m_{1,2} = \pm 2i$

$\Rightarrow$ General Solution: $y(x) = C_1 \cos(2x) + C_2 \sin(2x)$

BVP: $\left.\begin{aligned} y(0) &= C_1 = 0 \\ y(\pi) &= C_1 = 0 \end{aligned}\right\} \Rightarrow$ OK, so $C_1 = 0$, but C_2 can be anything!

$\Rightarrow$ ∞-many solutions: $\boxed{y = C \sin(2x)}$

What if BVP was:

$$y(0) = 0; \; y(\pi) = 1 \; ?$$

$\left.\begin{aligned} y(0) &= C_1 = 0 \\ y(\pi) &= C_1 = 1 \end{aligned}\right\} \Rightarrow 0 = 1$ Contradiction!

$\Rightarrow$ NO Solutions

II The substitutions in HW2 (9–11)

ODE's in the form $y' = F(ax+by+c)$, $b \neq 0$

Solved via the substitution $u := ax+by+c$ $\Rightarrow$ Separable ODE in u

Examples: $\boxed{\frac{dy}{dx} = (x+y+1)^2}$; $\boxed{\frac{dy}{dx} = 1+e^{y-x+5}}$

$u := x+y+1$

$u' = 1+y' \Rightarrow y' = u'-1$

Original Eqn. becomes: $u'-1 = u^2$

$$\frac{du}{dx} = u^2+1$$

$$\frac{1}{u^2+1}du = dx$$

$$\arctan(u) = x+C$$

$$\left.\begin{array}{l} u = \tan(x+C) \\ u = x+y+1 \end{array}\right\}$$

$\Rightarrow x+y+1 = \tan(x+C)$

$$\boxed{y = \tan(x+C)-x-1}$$

$u = y-x+5$

$u' = y'-1 \Rightarrow y' = u'+1$

ODE becomes:

$$u'+1 = 1+e^u$$

$$u' = e^u$$

$$\frac{du}{dx} = e^u$$

$$e^{-u}du = dx$$

$$-e^{-u} = x+C$$

$$u = -\ln(-x+C)$$

$$y-x+5 = -\ln(-x+C)$$

$$\boxed{y = x-5+\ln(-x+C)}$$