

3. Qualitative Methods; Autonomous Equations

• Qualitative Methods: determine properties of the solutions of a DE using geometrical methods (without actually solving the DE).

• Examples of properties investigated via qualitative methods:

- equilibrium solutions
- behavior of solutions near equilibrium points
- long-term behavior of solutions (limits at $\pm\infty$)
- monotonicity of solutions.

• Direction Fields: Consider a first order ODE: $\frac{dy}{dx} = f(x, y)$

Suppose $y(x)$ is a solution of this ODE on some interval $I \subset \mathbb{R}$.

Then $y(x)$ must be differentiable, and therefore continuous on I

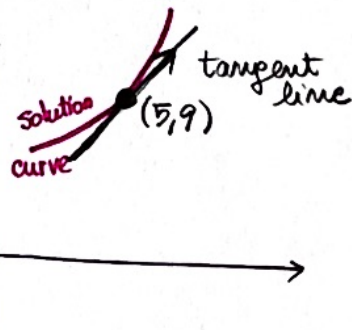
\Rightarrow at every point $(x, y(x))$, the curve has a tangent line whose slope is exactly $\frac{dy}{dx}(x) = f(x, y(x))$

$\Rightarrow \frac{dy}{dx} = f(x, y) \rightarrow$ slope of tangent line to the graph of a solution $y(x)$ passing through $(x, y(x))$.

(Ex): $\frac{dy}{dx} = \frac{1}{45}xy$

$$x=5, y=9 \Rightarrow \left. \frac{dy}{dx} \right|_{(5,9)} = 1$$

\Rightarrow tangent line at $(5, 9)$ has slope 1.



• Doing this systematically, i.e. evaluate $f(x, y)$ over a rectangular grid of points (x, y) and draw a small line element w/ the corresponding slope, oriented in the direction of increasing x , yields a direction field.