Inverse Laplace Transform

① $\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\} = \frac{1}{2}t^2$

② $\mathcal{L}^{-1}\left\{\frac{1}{s^2} - \frac{48}{s^5}\right\} = t - 48\,\frac{1}{4!}t^4 = t - 2t^4$

③ $\mathcal{L}^{-1}\left\{\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s-2}\right\} = t - 1 + e^{2t}$

④ $\mathcal{L}^{-1}\left\{\frac{(s+1)^3}{s^4}\right\} = \mathcal{L}^{-1}\left\{\frac{s^3+3s^2+3s+1}{s^4}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + 3\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} + 3\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s^4}\right\}$
$= 1 + 3t + \frac{3}{2}t^2 + \frac{1}{6}t^3.$

⑤ $\mathcal{L}^{-1}\left\{\frac{5}{s^2+49}\right\} = \frac{5}{7}\sin(7t).$

⑥ $\mathcal{L}^{-1}\left\{\frac{1}{s^2-16}\right\} = \frac{1}{4}\sinh(4t).$

⑦ $\mathcal{L}^{-1}\left\{\frac{1}{4s+1}\right\} = \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s+1/4}\right\} = \frac{1}{4}e^{-1/4\,t}$

⑧ $\mathcal{L}^{-1}\left\{\frac{4s}{4s^2+1}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{s^2+1/4}\right\} = \cos\left(\frac{1}{2}t\right).$

⑨ $\mathcal{L}^{-1}\left\{\frac{2s-6}{s^2+9}\right\} = 2\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\} - 6\mathcal{L}^{-1}\left\{\frac{1}{s^2+9}\right\} = 2\cos(3t) - 2\sin(3t).$

⑩ $\mathcal{L}^{-1}\left\{\frac{1}{s^2+3s}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s(s+3)}\right\} = \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\} = \frac{1}{3} - \frac{1}{3}e^{-3t}.$

$$\frac{1}{s(s+3)} = \frac{A}{s} + \frac{B}{s+3} \quad ; \quad 1 = A(s+3) + Bs$$

$s = 0 \Rightarrow \boxed{A = 1/3}$

$s = -3 \Rightarrow \boxed{B = -1/3}$

⑪ $\mathcal{L}^{-1}\left\{\frac{s}{s^2+2s-3}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{(s+3)(s-1)}\right\} = \frac{3}{4}\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\} + \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} = \frac{3}{4}e^{-3t} + \frac{1}{4}e^{t}$

$$\frac{s}{(s+3)(s-1)} = \frac{A}{s+3} + \frac{B}{s-1}$$

"Cover up" $(s-1)$: $\left.\frac{s}{s+3}\right|_{s=1} = B \Rightarrow \boxed{B = 1/4}$

"Cover up" $(s+3)$: $\left.\frac{s}{s-1}\right|_{s=-3} = A \Rightarrow \boxed{A = 3/4}$

12. $\mathcal{L}^{-1}\left\{\frac{s}{(s-2)(s-3)(s-6)}\right\} = \frac{1}{2}e^{2t} - e^{3t} + \frac{1}{2}e^{6t}$

$$\frac{s}{(s-2)(s-3)(s-6)} = \frac{A}{s-2} + \frac{B}{s-3} + \frac{C}{s-6}$$

$$\frac{s}{(s-3)(s-6)}\Big|_{s=2} = A \Rightarrow A = 1/2 \; ; \; \frac{s}{(s-2)(s-6)}\Big|_{s=3} = B \Rightarrow B = -1$$

$$\frac{s}{(s-2)(s-3)}\Big|_{s=6} = C \Rightarrow C = 1/2$$