1) $y' - 3y = \delta(t-2)$; $y(0)=0$

$$sY(s) - 3Y(s) = e^{-2s} \Rightarrow Y(s) = \frac{e^{-2s}}{s-3}$$

$$\Rightarrow y(t) = \mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s-3}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\}\Big|_{t\to t-2} u_2(t)$$

$$\boxed{y(t) = e^{3t-6}u_2(t)}$$

2) $y'' + y = \delta(t-2\pi)$; $y(0)=0$, $y'(0)=1$

$$s^2Y(s) - 1 + Y(s) = e^{-2\pi s}$$

$$Y(s) = \frac{1}{s^2+1} + \frac{e^{-2\pi s}}{s^2+1} \Rightarrow y(t) = \sin t + \sin(t-2\pi)u_{2\pi}(t)$$

$$\boxed{y(t) = \sin t\,(1 + u_{2\pi}(t))}$$

3) $y'' + y = \delta\left(t-\frac{\pi}{2}\right) + \delta\left(t-\frac{3\pi}{2}\right)$; $y(0) = y'(0) = 0.$

$$s^2Y(s) + Y(s) = e^{-\pi/2\, s} + e^{-3\pi/2\, s} \Rightarrow Y(s) = \frac{e^{-\pi/2\, s}}{s^2+1} + \frac{e^{-3\pi/2\, s}}{s^2+1}$$

$$\Rightarrow y(t) = \sin\left(t-\frac{\pi}{2}\right)u_{\pi/2}(t) + \sin\left(t-\frac{3\pi}{2}\right)u_{3\pi/2}(t)$$

$$\boxed{y(t) = -\cos(t)\,u_{\pi/2}(t) + \cos(t)\,u_{3\pi/2}(t)}$$

4) $y'' + 2y' = \delta(t-1)$; $y(0)=0$, $y'(0)=1.$

$$s^2Y(s) - 1 + 2sY(s) = e^{-s}$$

$$(s^2+2s)Y(s) = 1 + e^{-s} \Rightarrow Y(s) = \frac{1}{s^2+2s} + \frac{e^{-s}}{s^2+2s}$$

$$\boxed{y(t) = \frac{1}{2}(1-e^{-2t}) + \frac{1}{2}(1-e^{-2t+2})u_1(t)}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2+2s}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s(s+2)}\right\}$$

$$= \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{s+2}\right\}$$

$$= \frac{1}{2}(1-e^{-2t})$$

⑤ $y'' + 4y' + 5y = \delta(t-2\pi)\ ;\ y(0)=0,\ y'(0)=0$

$$s^2Y(s) + 4sY(s) + 5Y(s) = e^{-2\pi s} \Rightarrow Y(s) = \frac{e^{-2\pi s}}{s^2+4s+5}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2+4s+5}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2+1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\Big|_{s\to s+2}\right\}$$

$$= e^{-2t}\sin t$$

$$\boxed{y(t) = e^{-2t+4\pi}\sin(t)u_{2\pi}(t)}$$

⑥ $y'' + 4y' + 13y = \delta(t-\pi) + \delta(t-3\pi)\ ;\ y(0)=1,\ y'(0)=0.$

$$s^2Y(s) - s + 4sY(s) - 4 + 13Y(s) = e^{-\pi s} + e^{-3\pi s}$$

$$(s^2+4s+13)Y(s) = s+4+e^{-\pi s}+e^{-3\pi s} \Rightarrow Y(s) = \frac{s+4}{s^2+4s+13} + \frac{e^{-\pi s}}{s^2+4s+13} + \frac{e^{-3\pi s}}{s^2+4s+1}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2+4s+13}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2+9}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s^2+9}\Big|_{s\to s+2}\right\} = e^{-2t}\frac{1}{3}\sin(3t)$$

$$\mathcal{L}^{-1}\left\{\frac{s+4}{s^2+4s+13}\right\} = \mathcal{L}^{-1}\left\{\frac{s+2+2}{(s+2)^2+9}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\Big|_{s\to s+2}\right\} + 2\mathcal{L}^{-1}\left\{\frac{1}{s^2+9}\Big|_{s\to s+2}\right\}$$

$$= e^{-2t}\cos(3t) + \frac{2}{3}e^{-2t}\sin(3t).$$

$$\boxed{\begin{aligned} y(t) = e^{-2t}\cos(3t) + \frac{2}{3}e^{-2t}\sin(3t) \\ + e^{-2t+2\pi}\frac{1}{3}\sin(3t-3\pi)u_\pi(t) \\ + e^{-2t+6\pi}\frac{1}{3}\sin(3t-9\pi)u_{3\pi}(t) \end{aligned}}$$