Math 308 - Differential Equations **Homework 16**
Section 501
Texas A&M, Spring 2022

First-Order Linear Systems of Differential Equations with Constant Coefficients
I. Real, Distinct Eigenvalues

Solve the following systems:

1. $\begin{cases} \frac{dx}{dt} = -4x + y + z \\ \frac{dy}{dt} = x + 5y - z \\ \frac{dz}{dt} = y - 3z. \end{cases}$

2. $\mathbf{x}' = \begin{pmatrix} 10 & -5 \\ 8 & -12 \end{pmatrix} \mathbf{x}.$

3. $\mathbf{x}' = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \mathbf{x}.$

4. $\begin{cases} x'(t) = -4x + 2y \\ y'(t) = -\frac{5}{2}x + 2y. \end{cases}$

5. $\begin{cases} x'(t) = x + y - z \\ y'(t) = 2y \\ z'(t) = y - z. \end{cases}$

6. $\mathbf{x}' = \begin{pmatrix} -1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{pmatrix} \mathbf{x}.$

II. Complex Eigenvalues

Solve the following systems:

1. $\mathbf{x}' = \begin{pmatrix} 6 & -1 \\ 5 & 4 \end{pmatrix} \mathbf{x}.$

2. $\mathbf{x}' = \begin{pmatrix} 6 & -1 \\ 5 & 2 \end{pmatrix} \mathbf{x}.$

3. $\mathbf{x}' = \begin{pmatrix} 5 & 1 \\ -2 & 3 \end{pmatrix} \mathbf{x}.$

4. $\mathbf{x}' = \begin{pmatrix} 4 & -5 \\ 5 & -4 \end{pmatrix} \mathbf{x}.$

5. $\begin{cases} x'(t) = z \\ y'(t) = -z \\ z'(t) = y. \end{cases}$

6. $\mathbf{x}' = \begin{pmatrix} 1 & -1 & 2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \mathbf{x}.$

7. $\mathbf{x}' = \begin{pmatrix} 2 & 5 & 1 \\ -5 & -6 & 4 \\ 0 & 0 & 2 \end{pmatrix} \mathbf{x}.$

III. Repeated Eigenvalues

8. $\mathbf{x}' = \begin{pmatrix} 4 & -1 \\ 1 & 2 \end{pmatrix} \mathbf{x}.$

9. $\mathbf{x}' = \begin{pmatrix} 7 & 1 \\ -4 & 3 \end{pmatrix} \mathbf{x}.$

10. $\mathbf{x}' = \begin{pmatrix} -1 & 3/2 \\ -1/6 & -2 \end{pmatrix} \mathbf{x}.$

11. $\mathbf{x}' = \begin{pmatrix} 2 & 1/2 \\ -1/2 & 1 \end{pmatrix} \mathbf{x}.$

For each of the matrices below, find the eigenvalues and, for each eigenvalue, find its algebraic and its geometric multiplicity. If the matrix A is non-defective, solve the system $\mathbf{x}' = A\mathbf{x}$.

12. $A = \begin{pmatrix} 1 & -2 & 2 \\ -2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}.$

13. $A = \begin{pmatrix} 2 & 1 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 2 \end{pmatrix}.$

14. $A = \begin{pmatrix} 3 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}.$