

Homogeneous Linear ODEs with Constant Coefficients (1)

Find the general solution for the following differential equations:

1. $4y'' + y' = 0$.
2. $y'' - 36y = 0$.
3. $12y'' - 5y' - 2y = 0$.

Solve the IVP:

4. $y'' - y = 0$; $y(0) = y'(0) = 1$.

Solve the **BVP**:

5. $y'' - y = 0$; $y(0) = 1$; $y'(1) = 0$.

6. Find a homogeneous linear ODE which could have the solution:

$$y = 4e^{6x} + \pi e^{-2x}.$$

Hint: Look for a linear ODE with constant coefficients.

7. Given that $y_1 = x^3$ is a solution of the ODE:

$$x^2 y'' - 6y = 0,$$

use reduction of order to find a *second* (linearly independent) solution y_2 on the interval $(0, \infty)$, and then write the general solution.

- Set $y_2(x) = u(x)x^3$, where $u(x)$ is some function you will try to find.
- Find the condition u should satisfy in order for y_2 to be a solution.
- Use the substitution $v = u'$ to reduce your condition on u from a second order equation, to a first order equation. Solve the first order ODE in v (should be separable), then find u .
- Write the general solution $y = c_1 y_1 + c_2 y_2$.