

3.6 Variation of Parameters

Find the general solution for the following differential equations on the interval indicated:

1. $y'' + 3y' + 2y = \frac{1}{1 + e^x}; \quad x \in \mathbb{R}.$

2. $y'' + 3y' + 2y = \sin(e^x); \quad x \in \mathbb{R}.$

3. $y'' + 3y' + 2y = e^{-x}; \quad x \in \mathbb{R}.$

4. $y'' + 2y' + y = e^{-x} \ln(x); \quad x \in (0, \infty).$

5. $y'' - 2y' + y = \frac{e^x}{x}; \quad x \in (0, \infty).$

6. Given that

$$y_1 = x \text{ and } y_2 = x \ln(x)$$

form a fundamental set of solutions to

$$x^2 y'' - xy' + y = 0$$

on $x \in (0, \infty)$, find the general solution to

$$x^2 y'' - xy' + y = 4x \ln(x)$$

on $x \in (0, \infty)$.

7. Given that

$$y_1 = \frac{\cos(x)}{\sqrt{x}} \text{ and } y_2 = \frac{\sin(x)}{\sqrt{x}}$$

form a fundamental set of solutions to

$$x^2 y'' + xy' + (x^2 - \frac{1}{4})y = 0$$

on $x \in (0, \infty)$, find the general solution to

$$x^2 y'' + xy' + (x^2 - \frac{1}{4})y = x\sqrt{x}$$

on $x \in (0, \infty)$.