Math 308 - Differential Equations **Homework 9**
Section 501
Texas A&M, Spring 2022

Properties of The Laplace Transform

Laplace transforms of some basic functions:

$\mathscr{L}\{1\} = \dfrac{1}{s};\ s > 0.$

$\mathscr{L}\{t^n\} = \dfrac{n!}{s^{n+1}};\ s > 0.$

$\mathscr{L}\{e^{at}\} = \dfrac{1}{s-a};\ s > a.$

$\mathscr{L}\{\sin(at)\} = \dfrac{a}{s^2+a^2};\ s > 0.$

$\mathscr{L}\{\cos(at)\} = \dfrac{s}{s^2+a^2};\ s > 0.$

$\mathscr{L}\{\sinh(at)\} = \dfrac{a}{s^2-a^2};\ s > |a|.$

$\mathscr{L}\{\cosh(at)\} = \dfrac{s}{s^2-a^2};\ s > |a|.$

Properties of the Laplace transform:

Translation Theorem: $\boxed{\mathscr{L}\left\{e^{at}f(t)\right\} = F(s-a) = \mathscr{L}\{f(t)\}|_{s\to s-a}}$

Derivatives of Laplace Transforms: $\boxed{\mathscr{L}\left\{t^n f(t)\right\} = (-1)^n \dfrac{d^n}{ds^n}F(s)}$

Laplace Transforms of Derivatives: $\boxed{\mathscr{L}\{f^{(n)}(t)\} = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - sf^{(n-2)}(0) - f^{(n-1)}(0)}$

Find the Laplace transforms below, with the frequency domain as well.

1. $\mathscr{L}\{te^{10t}\}$.

2. $\mathscr{L}\{t^3e^{-2t}\}$.

3. $\mathscr{L}\{e^t\sin(3t)\}$.

4. $\mathscr{L}\{e^{-t}\sin^2 t\}$.

5. $\mathscr{L}\{t(e^t+e^{2t})^2\}$.

6. $\mathscr{L}\{t\cos(2t)\}$.

7. $\mathscr{L}\{e^{5t}\sinh(3t)\}$.

8. $\mathscr{L}\{te^{2t}\sin(6t)\}$.

9. $\mathscr{L}\{e^{-2t}(t^3+1)^2\}$.

10. $\mathscr{L}\{te^{at}\sin(bt)\}$.

11. Suppose y is the solution to the IVP:

$$y'' - 2y' + y = 0;\ y(0) = 2, y'(0) = 3.$$

Find $Y(s)$, the Laplace transform of y (without solving the IVP!).