Math 308 - Differential Equations **Homework 2**
Section 501
Texas A& M University, Spring 2022

Separable Equations

Solve the following differential equations using separation of variables:

1. $\dfrac{dy}{dx} = \sin(5x)$

2. $xy' = 4y$

3. $(4y + yx^2)\,dy - (2x + xy^2)\,dx = 0$

4. $\dfrac{dy}{dx} = \dfrac{xy + 3x - y - 3}{xy - 2x + 4y - 8}$

5. $y\,dy = 4x\sqrt{y^2 + 1}\,dx;\ \ y(0) = 1$

6. $\dfrac{dx}{dy} = 4(x^2 + 1);\ \ x\left(\dfrac{\pi}{4}\right) = 1$

7. $x^2y' = y - xy;\ \ y(-1) = -1$

8. $e^y \sin(2x)\,dx + \cos x(e^{2y} - y)\,dy = 0.$

Differential equations of the form $y'(x) = F(ax + by + c)$, with $b \neq 0$, can be reduced to a separable equation by making the substitution:

$$u = ax + by + c.$$

Differentiating both sides above, we have:

$$\frac{du}{dx} = a + b\frac{dy}{dx} \ \Rightarrow\ \frac{dy}{dx} = \frac{1}{b}\left(\frac{du}{dx} - a\right),$$

and the original equation may be rewritten as:

$$\frac{du}{dx} = a + bF(u).$$

Use this method to solve the equations:

9. $\dfrac{dy}{dx} = (x + y + 1)^2$

10. $\dfrac{dy}{dx} = 1 + e^{y-x+5}$

11. $\dfrac{dy}{dx} = 2 + \sqrt{y - 2x + 3}.$