NAME: Solutions

Math 2401 (K1-K3)
3/2/2015

Quiz 6 - Take-home

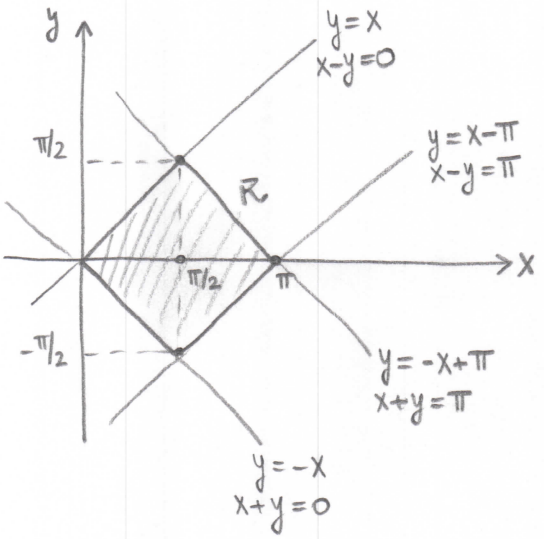
Instructions: You will have to turn in the completed quiz in recitation, Wednesday 3/4/2015. You are allowed to collaborate with one another, but the solutions you turn in must be your own - that is, you cannot copy another student's solution, but must instead write your own.

Let R be the region in the x, y-plane given by the square with vertices $(0,0)$, $(\pi/2, \pi/2)$, $(\pi/2, -\pi/2)$, and $(\pi, 0)$. Consider the transformation:

$$u = x + y; \quad v = x - y.$$

3pts. → a). Find and sketch the region G in the u, v-plane corresponding to R under the transformation above.
2pts. → b). Find the inverse transformation (i.e. find x, y in terms of u, v).
2 pts. → c). Find the Jacobian of this transformation.
3 pts. → d). Use this transformation to compute:

$$\iint_R (x-y)\sin(x+y)\,dA.$$

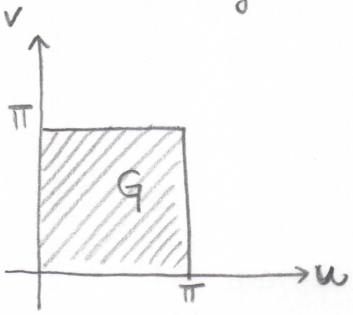


a). Boundaries of R:

$x-y=0$
$x-y=\pi$
$x+y=0$
$x+y=\pi$

Boundaries of G:

$v=0$
$v=\pi$
$u=0$
$u=\pi$



The region G is a square, with vertices $(0,0)$, $(\pi,0)$, $(0,\pi)$, (π,π). (3pts.)

b). $u = x+y$
$v = x-y$
$\oplus \quad u+v = 2x$

$u = x+y$
$v = x-y$
$\ominus \quad u-v = 2y$

$$\boxed{\begin{aligned} x &= \tfrac{1}{2}(u+v) \\ y &= \tfrac{1}{2}(u-v) \end{aligned}} \quad \text{(2pts.)}$$

c). $J = \begin{vmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{vmatrix} = -\frac{1}{4} - \frac{1}{4} = \boxed{-\frac{1}{2}}$ (2pts.)

d). $\iint_R (x-y)\sin(x+y)\,dA$
$= \iint_G v\sin(u)\,|J(u,v)|\,d(u,v)$ } (1 pt.)

$= \int_0^\pi \int_0^\pi \frac{1}{2} v\sin(u)\,dv\,du = \int_0^\pi \left(\frac{1}{4}v^2\sin(u)\Big|_{v=0}^{v=\pi}\right) du = \int_0^\pi \frac{\pi^2}{4}\sin(u)\,du$

$= -\frac{\pi^2}{4}\cos(u)\Big|_0^\pi = -\frac{\pi^2}{4}(-1-1) = \boxed{\frac{\pi^2}{2}}$

Computation - 1½ pts.
Final answer - ½ pt.