1. [20 points] Find the volume of the region in space bounded above by the surface:

$$z = xye^{xy^2},$$

and bounded below by the rectangle: $0 \le x \le \ln(7)$; $0 \le y \le 1$.

$$V = \int_0^{\ln(7)} \int_0^1 xye^{xy^2}\,dy\,dx$$

(4pts.) bounds (2pts.) setup (2pts.)

$$= \int_0^{\ln(7)} \frac{1}{2} e^{xy^2} \Big|_{y=0}^{y=1} dx$$

(4 pts.) - antiderivative

$$= \int_0^{\ln(7)} \frac{1}{2}(e^x - 1)\,dx$$

(3 pts.) - evaluation

$$= \frac{1}{2}(e^x - x) \Big|_{x=0}^{x=\ln(7)}$$

(4 pts.) - antiderivative

$$= \frac{1}{2}(7 - \ln(7) - 1 + 0)$$

(2 pts.) - evaluation

$$= \boxed{\frac{1}{2}(6 - \ln(7))}$$

(3 pts.) - final answer

2. [20 points] Find:

$$\int_1^2 \int_1^{\sqrt{z}} \int_{\ln(2y)}^{\ln(4y)} e^{x+y^2+z}\, dx\, dy\, dz.$$

$$\int_1^2 \int_1^{\sqrt{z}} \int_{\ln(2y)}^{\ln(4y)} e^x \cdot e^{y^2} \cdot e^z \, dx\, dy\, dz \qquad \text{(2pts.)}$$

$$= \int_1^2 \int_1^{\sqrt{z}} e^{y^2} \cdot e^z \, e^x \Big|_{x=\ln(2y)}^{x=\ln(4y)} dy\, dz \qquad \text{(4 pts.)}$$

$$= \int_1^2 \int_1^{\sqrt{z}} e^{y^2} \cdot e^z \,(2y)\, dy\, dz \qquad \text{(2 pts.)}$$

$$= \int_1^2 e^z \cdot e^{y^2} \Big|_{y=1}^{y=\sqrt{z}} dz \qquad \text{(5 pts.)}$$

$$= \int_1^2 e^z \cdot (e^z - e)\, dz \qquad \text{(2 pts.)}$$

$$= \int_1^2 (e^{2z} - e \cdot e^z)\, dz$$

$$= \left(\frac{1}{2} e^{2z} - e \cdot e^z\right)\Big|_1^2 \qquad \text{(3 pts.)}$$

$$= \frac{1}{2} e^4 - e \cdot e^2 - \frac{1}{2} e^2 + e^2$$

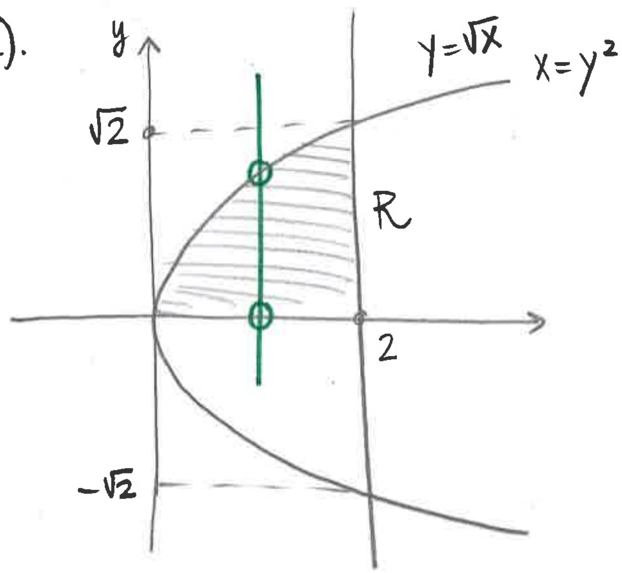
$$= \boxed{\frac{1}{2} e^4 - e^3 + \frac{1}{2} e^2} \qquad \text{(2 pts.)}$$

3. [20 points] Consider the integral:

$$\int_0^{\sqrt{2}} \int_{y^2}^{2} y^3 e^{x^3}\, dx\, dy.$$

a). Sketch the region of integration.
b). Compute the integral (you may want to switch the order of integration if you cannot compute it as given).

(a).



(5pts.)

(b). Take vertical cross-sections:

$$\int_0^2 \int_0^{\sqrt{x}} y^3 e^{x^3}\, dy\, dx \qquad \text{(5pts.)}$$

$$= \int_0^2 \frac{y^4}{4} e^{x^3} \Big|_{y=0}^{y=\sqrt{x}} dx \qquad \text{(4pts.)}$$

$$= \int_0^2 \frac{x^2}{4} e^{x^3} dx \qquad \text{(1pt.)}$$

$$= \frac{1}{4} \cdot \frac{1}{3} e^{x^3} \Big|_0^2 \qquad \text{(4pts.)}$$

$$= \boxed{\frac{1}{12}(e^8 - 1)} \qquad \text{(1pt.)}$$

4. [18 points] Find:

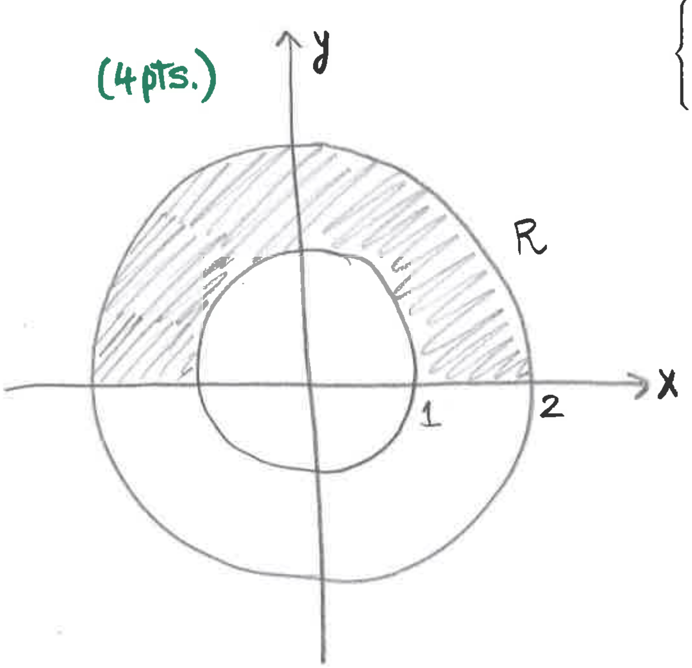
$$\iint_R \sin(x^2+y^2)\,dA,$$

where R is the region in the x, y-plane given by:

$$\begin{cases} 1 \le x^2+y^2 \le 4 \\ y \ge 0. \end{cases}$$

Sketch the region

(4pts.)



$$\underbrace{\int_0^{\pi}}_{(2\text{pts.})} \underbrace{\int_1^{2}}_{(2\text{pts.})} \underbrace{\sin(r^2)}_{(2\text{pts.})} \underbrace{r\,dr\,d\theta}_{(2\text{pts.})}$$

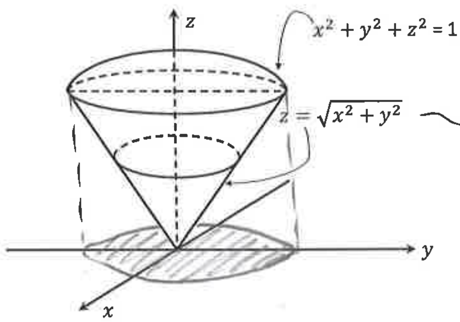
$$= \int_0^{\pi} -\frac{1}{2}\cos(r^2)\Big|_{r=1}^{r=2} d\theta \qquad (3\text{pts.})$$

$$= \int_0^{\pi} -\frac{1}{2}(\cos 4 - \cos 1)\,d\theta \qquad (1\text{pt.})$$

$$= -\frac{1}{2}(\cos 4 - \cos 1)\,\theta\Big|_0^{\pi} \qquad (1\text{pt.})$$

$$= \boxed{\frac{\pi}{2}(\cos 1 - \cos 4)} \qquad (1\text{pt.})$$

5. [16 points] Using *cylindrical coordinates*, set up the triple integral to compute the volume of the solid enclosed by the sphere $x^2 + y^2 + z^2 = 1$ and the cone $z = \sqrt{x^2 + y^2}$ (pictured below). You do not have to compute the value of the integral.



$x^2+y^2+z^2=1 \rightsquigarrow r^2+z^2=1 \Rightarrow z=\sqrt{1-r^2}$ (3pts.)

$z=\sqrt{x^2+y^2} \rightsquigarrow z=\sqrt{r^2} \Rightarrow z=r$ (3pts.)

Circle of intersection:

$$\begin{array}{r} x^2+y^2+z^2=1 \\ x^2+y^2-z^2=1 \\ \hline \ominus \quad 2z^2=1 \end{array}$$

$\Rightarrow z^2=1/2$ (3pts.)

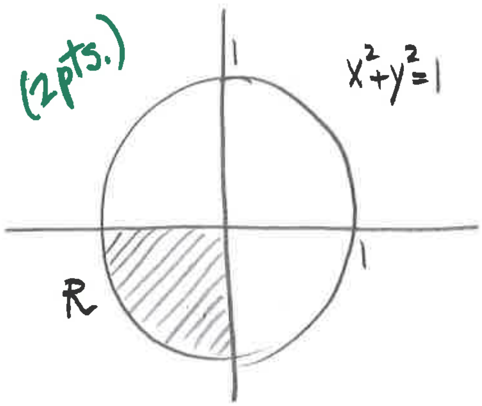
$\Rightarrow x^2+y^2=1/2 \quad r^2=1/2$

$$\boxed{V= \underbrace{\int_0^{2\pi}}_{1\text{ pt.}} \underbrace{\int_0^{1/\sqrt{2}}}_{2\text{ pts.}} \underbrace{\int_r^{\sqrt{1-r^2}}}_{2\text{ pts.}} \underbrace{r}_{1\text{ pt.}} \underbrace{dz\,dr\,d\theta}_{1\text{ pt.}}}$$

6. [6 points] Find: & Sketch

$$\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{0} \frac{5}{1+\sqrt{x^2+y^2}}\,dy\,dx.$$

(2pts.)



$$\int_{\pi}^{\frac{3\pi}{2}}\int_0^1 \frac{5}{1+r}\,r\,dr\,d\theta$$ (2pts.)

$$= \frac{5\pi}{2}\int_0^1 \frac{r}{1+r}\,dr$$

$$= \frac{5\pi}{2}\int_0^1 \frac{1+r-1}{1+r}\,dr$$

$$= \frac{5\pi}{2}\int_0^1 \left(1-\frac{1}{1+r}\right)dr$$

$$= \frac{5\pi}{2}\left(r-\ln(1+r)\right)\Big|_0^1$$

$$= \boxed{\frac{5\pi}{2}\left(1-\ln(2)\right)}$$

(2pts.)