

NAME: Solutions

Math 2401 (K1-K3)

Quiz 8

1. Find:

$$\int_C 2xy \, dx + 4y \, dy - yz \, dz$$

where C is the curve:

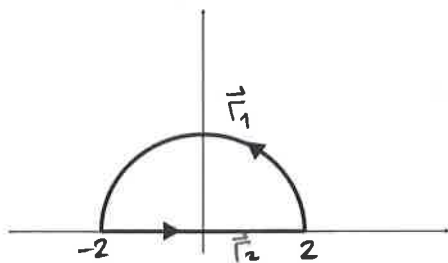
$$\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t\mathbf{k}, \quad 0 \leq t \leq 1.$$

2. Find the circulation and flux of the field $\mathbf{F} = x\mathbf{i} + y\mathbf{j}$ around the closed semicircular path that consists of the half-circle

$$\mathbf{r}_1(t) = 2\cos(t)\mathbf{i} + 2\sin(t)\mathbf{j}, \quad 0 \leq t \leq \pi,$$

followed by the line segment

$$\mathbf{r}_2(t) = t\mathbf{i}, \quad -2 \leq t \leq 2.$$



(4 pts.)

① $x=t; \, dx=dt$
 $y=t^2; \, dy=2t \, dt$
 $z=t; \, dz=dt$

$$\begin{aligned} \int_C 2xy \, dx + 4y \, dy - yz \, dz &= \\ &= \int_0^1 (2t^3 + 4t^2 \cdot 2t - t^3) \, dt = \int_0^1 9t^3 \, dt \\ &= \frac{9t^4}{4} \Big|_0^1 = \boxed{\frac{9}{4}} \end{aligned}$$

(3 pts.)

② Circulation: $\int_C \vec{F} \cdot d\vec{r} = \int_{C_1} \vec{F} \cdot d\vec{r} + \int_{C_2} \vec{F} \cdot d\vec{r} = 0 + 0 = \boxed{0}$

$$\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_1} M \, dx + N \, dy = \int_0^\pi 2\cos t (-2\sin t) + (2\sin t)(2\cos t) \, dt = \boxed{0}$$

$x = 2\cos t; \, dx = -2\sin t \, dt$
 $y = 2\sin t; \, dy = 2\cos t \, dt$
 $M = x; \, N = y$

$$\int_{C_2} \vec{F} \cdot d\vec{r} = \int_{C_2} M \, dx + N \, dy = \int_{-2}^2 t \, dt = \frac{t^2}{2} \Big|_{-2}^2 = \boxed{0}$$

$x = t; \, dx = dt$
 $y = 0; \, dy = 0$

(3pts)

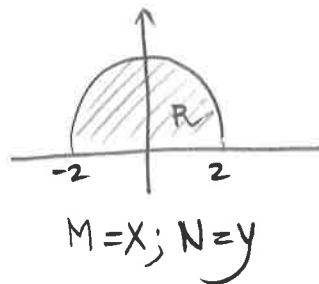
$$\begin{aligned}\text{Flux: } \int_{C_1} \vec{F} \cdot \vec{n} \, ds &= \int_{C_1} M \, dy - N \, dx \\ &= \int_0^\pi 2 \cos t (2 \cos t) - (2 \sin t)(-2 \sin t) \, dt \\ &= \int_0^\pi 4(\cos^2 t + \sin^2 t) \, dt \\ &= \boxed{4\pi}\end{aligned}$$

$$\begin{aligned}\int_{C_2} \vec{F} \cdot \vec{n} \, ds &= \int_{C_2} M \, dy - N \, dx \\ &= \int_{-2}^2 (t \cdot 0 - 0) \, dt = \boxed{0}\end{aligned}$$

$$\oint_C \vec{F} \cdot \vec{n} \, ds = \boxed{4\pi}$$

OR, using Green's Theorem :

$$\begin{aligned}\oint_C \vec{F} \cdot d\vec{r} &= \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dA \\ &= \iint_R (0 - 0) \, dA = \boxed{0}\end{aligned}$$



$$\begin{aligned}\oint_C \vec{F} \cdot \vec{n} \, ds &= \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) dA \\ &= \iint_R (1 + 1) \, dA = 2 \text{ Area}(R) = 2 \cdot \frac{\pi \cdot 2^2}{2} = \boxed{4\pi}\end{aligned}$$