Name: Solutions

March 23rd, 2015.
Math 2401; Sections K1, K2, K3.
Georgia Institute of Technology
Exam 3

I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech community. By signing my name below I pledge that I have neither given nor received help on this exam.

Pledged: ______________________

Problem	Possible Score	Earned Score
1	20	
2	20	
3	20	
4	18	
5	16	
6	6	
Total	100	

Remember that you must SHOW YOUR WORK to receive credit!

Good luck!

1. [20 points] Compute the double integral:

$$\iint_R y\sin(xy)\,dA,$$

where R is the rectangle in the xy-plane given by $1 \le x \le 2$; $0 \le y \le \pi$.

$$\int_0^{\pi}\int_1^2 y\sin(xy)\,dx\,dy = \int_0^{\pi} -\cos(xy)\Big|_{x=1}^{x=2}\,dy$$ (3pts. – antiderivative)

(6 pts. – setup)

$$= \int_0^{\pi}\left(-\cos(2y)+\cos(y)\right)dy$$ (3pts. – evaluation)

$$= \left(-\frac{1}{2}\sin(2y)+\sin y\right)\Big|_0^{\pi}$$ (6pts. – antiderivatives)

$$= -\frac{1}{2}\sin(2\pi)+\frac{1}{2}\sin(0)+\sin(\pi)-\sin(0)$$

$$= \boxed{0}$$ (2pts. – final answer)

2. [20 points] Find:

$$\int_1^2 \int_1^{\sqrt{z}} \int_{\ln(x)}^{\ln(3x)} e^{x^2+y+z}\, dy\, dx\, dz.$$

$$\int_1^2 \int_1^{\sqrt{z}} \int_{\ln(x)}^{\ln(3x)} e^{x^2} \cdot e^{y} \cdot e^{z}\, dy\, dx\, dz \qquad (2\text{ pts.})$$

$$= \int_1^2 \int_1^{\sqrt{z}} e^{x^2} \cdot e^{z} \cdot e^{y} \Big|_{y=\ln(x)}^{y=\ln(3x)}\, dx\, dz \qquad (4\text{ pts.})$$

$$= \int_1^2 \int_1^{\sqrt{z}} e^{x^2} \cdot e^{z}\,(3x - x)\, dx\, dz$$

$$= \int_1^2 \int_1^{\sqrt{z}} e^{x^2} \cdot e^{z}\,(2x)\, dx\, dz \qquad (2\text{ pts.})$$

$$= \int_1^2 e^{z} \cdot e^{x^2} \Big|_{x=1}^{x=\sqrt{z}}\, dz \qquad (5\text{ pts.})$$

$$= \int_1^2 e^{z} \cdot \left(e^{z} - e\right) dz \qquad (2\text{ pts.})$$

$$= \int_1^2 \left(e^{2z} - e^{z+1}\right) dz$$

$$= \left(\frac{1}{2}e^{2z} - e^{z+1}\right)\Big|_1^2 \qquad (3\text{ pts.})$$

$$= \frac{1}{2}e^4 - \frac{1}{2}e^2 - e^3 + e^2$$

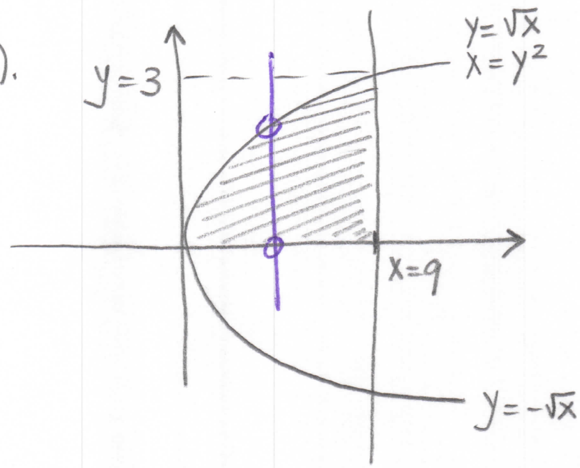
$$= \boxed{\frac{1}{2}e^4 - e^3 + \frac{1}{2}e^2} \qquad (2\text{ pts.})$$

3. [20 points] Consider the integral:

$$\int_0^9 \int_{y^2}^9 y\cos(x^2)\,dx\,dy.$$

a). Sketch the region of integration.
b). Compute the integral (you may want to switch the order of integration if you cannot compute it as given).

(a).



(5pts.)

(b). Vertical cross-sections:

$$\int_0^9 \int_0^{\sqrt{x}} y\cos(x^2)\,dy\,dx = \int_0^9 \frac{y^2}{2}\cos(x^2)\Big|_{y=0}^{y=\sqrt{x}}\,dx \quad \text{(4 pts.)}$$

(5pts.)

$$= \int_0^9 \frac{x}{2}\cos(x^2)\,dx \quad \text{(1 pt.)}$$

$$= \frac{1}{4}\sin(x^2)\Big|_{x=0}^{9} \quad \text{(4 pts.)}$$

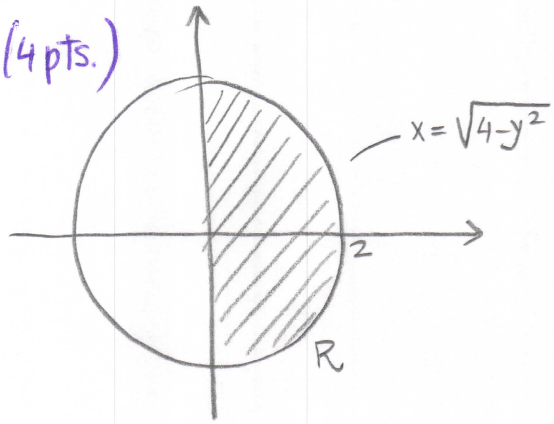
$$= \boxed{\frac{1}{4}\sin(81)} \quad \text{(1 pt.)}$$

4. [18 points] Sketch the region of integration and compute the integral:

$$\iint_R e^{-x^2-y^2}\,dA,$$

where R is the region in the x, y-plane bounded by the semicircle $x = \sqrt{4-y^2}$ and the y-axis.

(4 pts.)



$$\underbrace{\int_{-\pi/2}^{\pi/2}}_{2\text{pts.}} \underbrace{\int_0^2}_{2\text{pts.}} \underbrace{e^{-r^2}}_{2\text{pts.}} \underbrace{r\,dr\,d\theta}_{2\text{pts.}}$$

$$= \int_{-\pi/2}^{\pi/2} -\frac{1}{2}e^{-r^2}\Big|_{r=0}^{r=2}\,d\theta \qquad (3\text{pts.})$$

$$= \int_{-\pi/2}^{\pi/2} \left(-\frac{1}{2}e^{-4}+\frac{1}{2}\right)d\theta \qquad (1\text{pt.})$$

$$= \frac{1}{2}(1-e^{-4})\,\theta\Big|_{-\pi/2}^{\pi/2} \qquad (1\text{pt.})$$

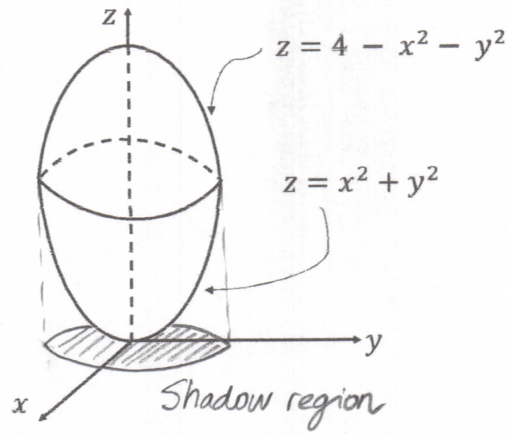
$$= \boxed{\frac{\pi}{2}(1-e^{-4})} \qquad (1\text{pt.})$$

5. [16 points] Using *cylindrical coordinates*, set up the triple integral to compute the volume of the solid enclosed by the two paraboloids:

$$z = 4 - x^2 - y^2;$$

$$z = x^2 + y^2,$$

pictured below. You do not have to compute the value of the integral.



$z = 4 - r^2$ (3pts.)

$z = r^2$ (3pts.)

Circle of intersection: $\begin{cases} z = 4 - r^2 \\ z = r^2 \end{cases}$

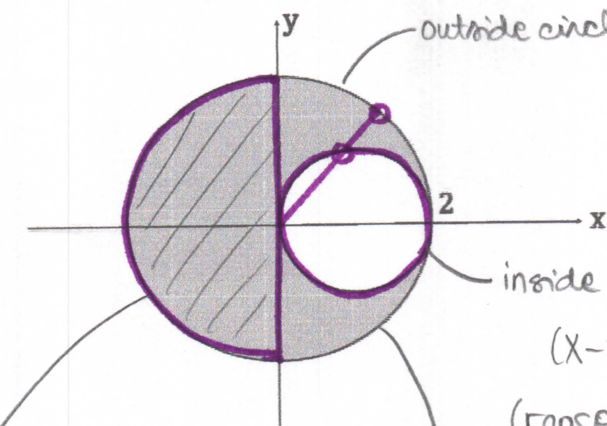
$$4 - r^2 = r^2$$
$$4 = 2r^2$$
$$2 = r^2$$
$$r = \sqrt{2}$$ (3pts.)

$$V = \underbrace{\int_0^{2\pi}}_{\text{1pt.}} \underbrace{\int_0^{\sqrt{2}}}_{\text{2pts.}} \underbrace{\int_{r^2}^{4-r^2}}_{\text{2pts.}} \underbrace{r}_{\text{1pt.}} \, \underbrace{dz\,dr\,d\theta}_{\text{1pt.}}$$

6. [6 points] Use polar coordinates to set up iterated integral(s) to evaluate:

$$\iint_R \sqrt{x^2+y^2}\,dA,$$

where R is the region in the xy-plane pictured (shaded) below, where both curves bounding the region are circles. You do not need to compute the value of the integral.
Hint: You may want to treat the right and left parts of the region separately, which would lead to two iterated integrals.



outside circle: $r=2$

inside circle: center at $(1,0)$, radius 1

$$(x-1)^2+y^2=1$$

$$(r\cos\theta-1)^2+(r\sin\theta)^2=1$$

$$r^2\cos^2\theta-2r\cos\theta+\cancel{1}+r^2\sin^2\theta=\cancel{1}$$

$$r^2=2r\cos\theta \qquad \boxed{r=2\cos\theta}$$ (2pts.)

left region:

$$\int_{\pi/2}^{3\pi/2}\int_0^2 r\cdot r\,dr\,d\theta$$

right region:

$$\int_{-\pi/2}^{\pi/2}\int_{2\cos\theta}^{2} r\cdot r\,dr\,d\theta$$

$$\underbrace{\int_{\pi/2}^{3\pi/2}}_{1/2\text{pt.}}\underbrace{\int_0^2}_{1/2\text{pt.}}\underbrace{r^2}_{(1\text{pt.})}\,dr\,d\theta+\underbrace{\int_{-\pi/2}^{\pi/2}}_{1/2\text{pt.}}\underbrace{\int_{2\cos\theta}^{2}}_{1\,1/2\text{pts.}} r^2\,dr\,d\theta$$