Worksheet 5 - Solutions

4) $f(x,y,z) = xyz$

$C: \vec{r}(t) = \cos(t)\,\vec{i} + \sin(t)\,\vec{j} + 3t\,\vec{k}, \quad 0 \le t \le 4\pi.$

$\vec{v}(t) = \langle -\sin t, \cos t, 3 \rangle \; ; \quad |\vec{v}(t)| = \sqrt{\sin^2 t + \cos^2 t + 9} = \sqrt{10}$

$$\int_C f\,ds = \int_0^{4\pi} \cos(t)\sin(t) \cdot 3t \cdot \sqrt{10}\,dt$$

$$= 3\sqrt{10} \int_0^{4\pi} t \sin(t)\cos(t)\,dt$$

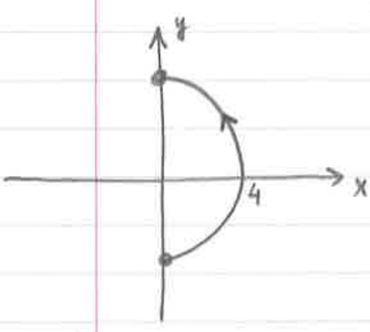
$$= 3\sqrt{10} \int_0^{4\pi} t \cdot \frac{1}{2}\sin(2t)\,dt$$

$$= \frac{3\sqrt{10}}{2} \int_0^{4\pi} t \cdot \left(\cos(2t)\right)'\,dt \left(\frac{-1}{2}\right)$$

$$= \frac{-3\sqrt{10}}{4}\left(t\cos(2t)\Big|_0^{4\pi} - \int_0^{4\pi} \cos(2t)\,dt \right.$$

$$= \frac{-3\sqrt{10}}{4}\left(4\pi - 0 - \underbrace{\frac{1}{2}\sin(2t)\Big|_0^{4\pi}}_{0}\right) = \boxed{-3\sqrt{10}\,\pi}$$

1) $\int_C xy^4\,ds$, C: right half of circle $x^2+y^2=16$



$x = 4\cos t$

$y = 4\sin t$

$-\pi/2 \le t \le \pi/2$

$\vec{r}(t) = \langle 4\cos t, 4\sin t \rangle$

$\vec{v}(t) = \langle -4\sin t, 4\cos t \rangle$

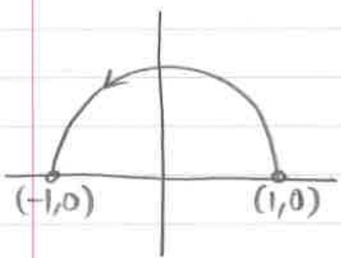
$|\vec{v}(t)| = \sqrt{16\sin^2 t + 16\cos^2 t} = \sqrt{16} = 4.$

$$\int_C xy^4\,ds = \int_{-\pi/2}^{\pi/2} 4\cos t\,(4\sin t)^4 \cdot 4\,dt$$

$$= 4^6\left(\frac{\sin^5 t}{5}\right)\Bigg|_{-\pi/2}^{\pi/2} = 4^6 \cdot \left(\frac{1}{5} - \left(-\frac{1}{5}\right)\right) = \frac{2 \cdot 4^6}{5}$$

$$= \boxed{\frac{8192}{5}}$$

(2) $\int_C (2+x^2y)\,ds$, C: upper half of unit circle $x^2+y^2=1$.



$\vec{r}(t) = \langle \cos t, \sin t \rangle$ $\qquad$ $\vec{v}(t) = \langle -\sin t, \cos t \rangle$

$0 \le t \le \pi$ $\qquad$ $|\vec{v}(t)| = 1$

$$\int_C (2+x^2y)\,ds = \int_0^{\pi} \left(2 + \cos^2 t \sin t\right) dt$$

$$= 2t\Big|_0^{\pi} - \frac{\cos^3 t}{3}\Big|_0^{\pi}$$

$$= 2\pi - \left(\frac{(-1)^3}{3} - \frac{1}{3}\right) = \boxed{2\pi + 2/3}$$

(3) $\int_C \frac{x^2}{y^{4/3}}\,ds$; C: $\vec{r}(t) = \langle t^2, t^3 \rangle$, $-3 \le t \le 1$.

$\vec{v}(t) = \langle 2t, 3t^2 \rangle$; $|\vec{v}(t)| = \sqrt{4t^2+9t^4} = \sqrt{t^2(4+9t^2)}$

$= |t|\sqrt{4+9t^2}$

$$\int_C \frac{x^2}{y^{4/3}}\,ds = \int_{-3}^{1} \underbrace{\frac{(t^2)^2}{(t^3)^{4/3}}}_{=1} |t|\sqrt{4+9t^2}\,dt$$

$$= \int_{-3}^{0} (-t)\sqrt{4+9t^2}\,dt + \int_0^1 t\sqrt{4+9t^2}\,dt$$

$$= -\frac{1}{9}\cdot\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}\left(4+9t^2\right)^{3/2}\Big|_{-3}^{0} + \frac{1}{9}\cdot\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}\left(4+9t^2\right)^{3/2}\Big|_0^1$$

$$= \frac{-1}{27}\left(4^{3/2} - 85^{3/2}\right) + \frac{1}{27}\left(13^{3/2} - 4^{3/2}\right)$$

$$= \frac{1}{27}\left(85\sqrt{85} - 8 + 13\sqrt{13} - 8\right)$$

$$= \boxed{\frac{1}{27}\left(85\sqrt{85} + 13\sqrt{13} - 16\right)}$$