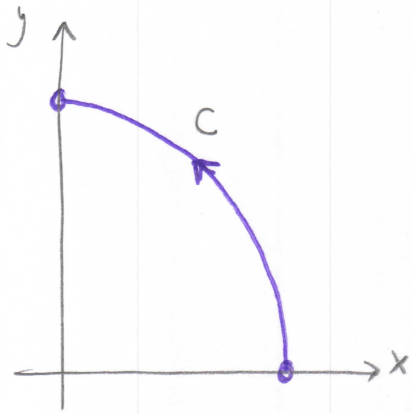


$$\textcircled{1} \vec{F}(x, y) = \langle x^2, -xy \rangle$$

$$C: \vec{r}(t) = \langle \cos t, \sin t \rangle, 0 \leq t \leq \pi/2$$



$$W = \int_C \vec{F} \cdot d\vec{r}$$

$$= \int_C M dx + N dy = \int_C x^2 dx - xy dy$$

$$= \int_0^{\pi/2} (\cos^2 t)(-\sin t) - (\cos t \sin t) \cos t dt$$

$$= \int_0^{\pi/2} -2 \cos^2 t \sin t dt$$

$$= \frac{2}{3} \cos^3 t \Big|_0^{\pi/2} = \frac{2}{3} (0 - 1) = \boxed{-\frac{2}{3}}$$

$$x = \cos t; dx = -\sin t dt$$

$$y = \sin t; dy = \cos t dt$$

$$\textcircled{2} \vec{F}(x, y, z) = \langle xy, yz, zx \rangle$$

$$C: \vec{r}(t) = \langle t, t^2, t^3 \rangle, 0 \leq t \leq 1$$

$$\int_C \vec{F} \cdot d\vec{r} = \int_C xy dx + yz dy + zx dz$$

$$= \int_0^1 (t^3 + t^5 \cdot 2t + t^4 \cdot 3t^2) dt$$

$$= \int_0^1 (t^3 + 5t^6) dt$$

$$= \left(\frac{t^4}{4} + \frac{5t^7}{7} \right) \Big|_0^1 = \frac{1}{4} + \frac{5}{7} = \boxed{\frac{27}{28}}$$

$$x = t; dx = dt$$

$$y = t^2; dy = 2t dt$$

$$z = t^3; dz = 3t^2 dt$$

$$\textcircled{3} \vec{F}(x,y) = \langle x^2 y^3, -y\sqrt{x} \rangle$$

$$C: \vec{r}(t) = \langle t^2, -t^3 \rangle, 0 \leq t \leq 1$$

$$x = t^2 ; dx = 2t dt$$

$$y = -t^3 ; dy = -3t^2 dt$$

$$\int_C \vec{F} \cdot d\vec{r} = \int_C x^2 y^3 dx - y\sqrt{x} dy$$

$\swarrow \sqrt{t^2} = t$ because $0 \leq t \leq 1$!

$$= \int_0^1 [t^4 (-t^9)(2t) - (-t^3) t (-3t^2)] dt$$

$$= \int_0^1 (-2t^{14} - 3t^6) dt$$

$$= \left(-\frac{2}{15} t^{15} - \frac{3}{7} t^7 \right) \Big|_0^1 = -\frac{2}{15} - \frac{3}{7} = \left(-\frac{59}{105} \right)$$

$$\textcircled{4} \vec{F}(x,y,z) = \langle \sin x, \cos y, xz \rangle$$

$$C: \vec{r}(t) = \langle t^3, -t^2, t \rangle ; 0 \leq t \leq 1$$

$$x = t^3 ; dx = 3t^2 dt$$

$$y = -t^2 ; dy = -2t dt$$

$$z = t ; dz = dt$$

$$\int_C \vec{F} \cdot d\vec{r} = \int_C \sin x dx + \cos y dy + xz dz$$

$$= \int_0^1 (\sin(t^3)(3t^2) + \cos(-t^2)(-2t) + t^3 \cdot t \cdot 1) dt$$

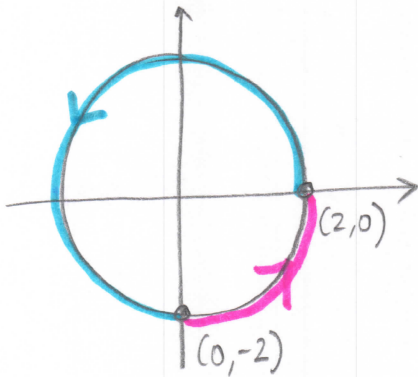
$$= \left(-\cos(t^3) + \sin(-t^2) + \frac{1}{5} t^5 \right) \Big|_0^1$$

$$= -\cos(1) + \sin(-1) + \frac{1}{5} - \left(\underbrace{-\cos(0)}_{-1} + \sin(0) + 0 \right)$$

$$= \boxed{\frac{6}{5} - \cos(1) - \sin(1)}$$

⑤ $\vec{F}(x,y) = \langle x-y, xy \rangle$

C_1 : arc of circle $x^2+y^2=4$ traversed counterclockwise from $(2,0)$ to $(0,-2)$



$C_1: \vec{r}(t) = \langle 2\cos t, 2\sin t \rangle, 0 \leq t \leq \frac{3\pi}{2}$

$x = 2\cos t;$
 $dx = -2\sin t dt$
 $y = 2\sin t;$
 $dy = 2\cos t dt$

$$\begin{aligned} \int_{C_1} \vec{F} \cdot d\vec{r} &= \int_{C_1} (x-y)dx + (xy)dy \\ &= \int_0^{3\pi/2} (2\cos t - 2\sin t)(-2\sin t) \\ &\quad + 4\sin t \cos t (2\cos t) dt \\ &= \int_0^{3\pi/2} (-4\sin t \cos t + 4\sin^2 t + 8\sin t \cos^2 t) dt \\ &= \left(2\cos^2 t + 2t - \sin(2t) - \frac{8}{3}\cos^3 t \right) \Big|_0^{3\pi/2} \\ &= (3\pi) - \left(2 - \frac{8}{3} \right) = \boxed{3\pi + \frac{2}{3}} \end{aligned}$$

$$\begin{aligned} &\int \sin^2 t dt \\ &= \int \frac{1 - \cos(2t)}{2} dt \\ &= \frac{t}{2} - \frac{1}{4}\sin(2t) \end{aligned}$$

C_2 : arc of the circle $x^2+y^2=4$ traversed counterclockwise from $(0,-2)$ to $(2,0)$:

$C_2: \vec{r}(t) = \langle 2\cos t, 2\sin t \rangle ; \boxed{-\frac{\pi}{2} \leq t \leq 0}$ or $\boxed{\frac{3\pi}{2} \leq t \leq 2\pi}$

$$\begin{aligned} \int_{C_2} \vec{F} \cdot d\vec{r} &= \int_{-\pi/2}^0 (x-y)dx + (xy)dy \\ &= \dots \text{same integral} \\ &= \left(2\cos^2 t + 2t - \sin(2t) - \frac{8}{3}\cos^3 t \right) \Big|_{-\pi/2}^0 \text{ or } \Big|_{3\pi/2}^{2\pi} \\ &= \left(2 - \frac{8}{3} \right) - (-\pi) = \boxed{\pi - \frac{2}{3}} \end{aligned}$$

Careful here and don't take $\int_{3\pi/2}^0 \dots dt$
 \rightarrow this is really $-\int_0^{3\pi/2} \dots dt$

\rightarrow doesn't matter which one

(c). Circulation of \vec{F} along circle $x^2+y^2=4$ (C):

$$\oint_C \vec{F} \cdot d\vec{r} = \int_{C_1} \vec{F} \cdot d\vec{r} + \int_{C_2} \vec{F} \cdot d\vec{r} = 3\pi + \frac{2}{3} + \pi - \frac{2}{3} = \boxed{4\pi}$$

(d). Flux of \vec{F} about C?

$$C: \vec{r}(t) = \langle 2\cos t, 2\sin t \rangle, 0 \leq t \leq 2\pi$$

$$\int_C \vec{F} \cdot \vec{n} \, ds = \int_C M \, dy - N \, dx$$

$$= \int_0^{2\pi} (2\cos t - 2\sin t)(2\cos t) - (4\sin t \cos t)(-2\sin t) \, dt$$

$$= \int_0^{2\pi} (4\cos^2 t - 4\sin t \cos t + 8\sin^2 t \cos t) \, dt$$

$$= \left(2t + \sin(2t) + 2\cos^2 t + \frac{8}{3} \sin^3 t \right) \Big|_0^{2\pi}$$

$$= \boxed{4\pi}$$

$$\int \cos^2 t \, dt$$

$$= \int \frac{1 + \cos(2t)}{2} \, dt$$

$$= \frac{t}{2} + \frac{1}{4} \sin(2t)$$

(6) $\vec{F}(x,y) = \langle e^{x-1}, xy \rangle$
 $C: \vec{r}(t) = \langle t^2, t^3 \rangle; 0 \leq t \leq 1$

$$x = t^2; \quad dx = 2t \, dt$$

$$y = t^3; \quad dy = 3t^2 \, dt$$

$$\int_C \vec{F} \cdot d\vec{r} = \int_C e^{x-1} \, dx + xy \, dy$$

$$= \int_0^1 (e^{t^2-1} \cdot 2t + t^5 \cdot 3t^2) \, dt$$

$$= \int_0^1 (e^{t^2-1} \cdot 2t + 3t^7) \, dt$$

$$= \left(e^{t^2-1} + \frac{3}{8} t^8 \right) \Big|_0^1$$

$$= 1 + \frac{3}{8} - \frac{1}{e} = \boxed{\frac{11}{8} - \frac{1}{e}}$$

$$\textcircled{7} \quad \vec{F}(x,y) = \langle x, y+2 \rangle$$

$$C: \vec{r}(t) = \langle t - \sin t, 1 - \cos t \rangle; \quad 0 \leq t \leq 2\pi$$

$$\text{Work} = \int_C \vec{F} \cdot d\vec{r}$$

$$= \int_C M dx + N dy$$

$$= \int_0^{2\pi} \left((t - \sin t)(1 - \cos t) + (3 - \cos t)(\sin t) \right) dt$$

$$= \int_0^{2\pi} \left(t - t \cos t - \sin t + \sin t \cos t + 3 \sin t - \cos t \sin t \right) dt$$

$$= \int_0^{2\pi} (t - t \cos t + 2 \sin t) dt$$

$$= \left(\frac{t^2}{2} - t \sin t - \cos t - 2 \cos t \right) \Big|_0^{2\pi}$$

$$= \frac{(2\pi)^2}{2} = \boxed{2\pi^2}$$

$$x = t - \sin t; \quad dx = (1 - \cos t) dt$$

$$y = 1 - \cos t; \quad dy = \sin t dt$$

$$\int t \cos t dt$$

$$= \int t (\sin t)' dt$$

$$= t \sin t - \int \sin t dt$$

$$= t \sin t + \cos t$$

$$\textcircled{8} \quad \vec{F}(x,y) = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$$

$$C: \vec{r}(t) = \langle \cos t, \sin t \rangle, \quad 0 \leq t \leq 2\pi$$

$$\begin{aligned} \text{Flux: } \oint_C \vec{F} \cdot \vec{n} \, ds &= \oint_C M \, dy - N \, dx \\ &= \int_0^{2\pi} [(-\sin t)(\cos t) - (\cos t)(-\sin t)] \, dt \\ &= \int_0^{2\pi} 0 \, dt = \boxed{0} \end{aligned}$$

$$\begin{aligned} x &= \cos t & dx &= -\sin t \, dt \\ y &= \sin t & dy &= \cos t \, dt \end{aligned}$$

$$M = \frac{-y}{x^2+y^2} = \frac{-\sin t}{\sin^2 t + \cos^2 t} = -\sin t$$

$$N = \frac{x}{x^2+y^2} = \frac{\cos t}{\cos^2 t + \sin^2 t} = \cos t$$

$$\begin{aligned} \text{Circulation: } \oint_C \vec{F} \cdot d\vec{r} &= \oint_C M \, dx + N \, dy \\ &= \int_0^{2\pi} ((-\sin t)(-\sin t) + (\cos t)(\cos t)) \, dt \\ &= \int_0^{2\pi} 1 \, dt = \boxed{2\pi} \end{aligned}$$