Math 2401 (D1-D3)
8/18/2014

Worksheet 1 - Review

1. Find all the values of x where the tangent lines to $y = \frac{x^3}{3} + 5x$ and $y = 3x^2$ are parallel.

Recall: - Slope of tangent line to a graph is given by the derivative
- Two lines are parallel when their slopes are equal

$y = \frac{x^3}{3} + 5x \Rightarrow y' = x^2 + 5$

$y = 3x^2 \Rightarrow y' = 6x$

$x^2 + 5 = 6x$

$x^2 - 6x + 5 = 0$

$(x-5)(x-1) = 0 \Rightarrow \boxed{x = 1, 5}$

2. Given that $f'(x) = 2e^x - 3$, and that $f(0) = 7$, find $f(x)$.

$f(x) = \int (2e^x - 3)\,dx = 2e^x - 3x + C$

$$\left.\begin{array}{r} f(0) = 2e^0 - 3\cdot 0 + C = 2 + C \\ f(0) = 7 \end{array}\right\} \Rightarrow C = 5 \Rightarrow \boxed{f(x) = 2e^x - 3x + 5}$$

3. Find $\frac{dy}{dx}$ for $y = x^{3x}$.

$y = x^{3x} = \left(e^{\ln(x)}\right)^{3x} = e^{\ln(x)\cdot 3x}$

$y' = e^{\ln(x)\cdot 3x}\left[\ln(x)\cdot 3x\right]' = x^{3x}\left[\frac{1}{x}\cdot 3x + \ln(x)\cdot 3\right]$

$= \boxed{x^{3x}\left[3 + 3\ln(x)\right]}$

4. For what value of c is the function below continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx+4 & \text{, if } x<1 \\ x^2+2c & \text{, if } x \geq 1 \end{cases}$$

Recall : A function is continuous at a point a in the domain if and only if
$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (cx+4) = c+4$$
$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (x^2+2c) = 1+2c = f(1)$$

$$c+4 = 1+2c \implies \boxed{c=3}$$

5. Find the derivative of $f(x) = \arctan(\sin(x))$.

$$f'(x) = \frac{\cos(x)}{1+\sin^2(x)}$$

Chain Rule

6. Find the derivative of $f(x) = 3^{x^2-3x+4}$.

$$f'(x) = 3^{x^2-3x+4} \cdot \ln(3) \cdot (2x-3)$$

Chain Rule

7. Find the integral:

$$\int_1^{e^4} \frac{1}{2x\sqrt{\ln(x)}}dx = \int_0^4 \frac{1}{2\sqrt{u}}du = \sqrt{u}\Big|_0^4 = \boxed{2}$$

$u = \ln(x)$
$du = \frac{1}{x}dx$

Substitution

$x=1 \implies u=\ln(1)=0$
$x=e^4 \implies u=\ln(e^4)=4$

8. Find the integral:

$$\int \frac{x-2}{x^2-6x+10}dx = \int \frac{x-2}{(x-3)^2+1}dx = \int \frac{(x-3)+1}{(x-3)^2+1}dx$$
$$= \int \frac{x-3}{(x-3)^2+1}dx + \int \frac{1}{(x-3)^2+1}dx = \boxed{\tfrac{1}{2}\ln(x^2-6x+10) + \arctan(x-3)+C}$$

$u = (x-3)^2+1$
$du = 2(x-3)dx$
$\frac{1}{2}du = (x-3)dx$

$$\int \frac{1}{u}\cdot\frac{1}{2}du$$
$$= \frac{1}{2}\ln|u| = \frac{1}{2}\ln|(x-3)^2+1| + C$$
$$= \frac{1}{2}\ln(x^2-6x+10)+C$$

9. Find the integral:

$$\int x^6 \ln(x)dx = \int\left(\frac{x^7}{7}\right)' \ln(x)dx = \frac{x^7}{7}\ln(x) - \int \frac{x^7}{7}(\ln(x))'dx$$
$$= \frac{x^7}{7}\ln(x) - \int \frac{x^7}{7}\cdot\frac{1}{x}dx = \frac{x^7}{7}\ln(x) - \frac{1}{7}\int x^6 dx$$
$$= \frac{x^7}{7}\ln(x) - \frac{1}{7}\cdot\frac{1}{7}x^7 + C$$
$$= \boxed{\frac{x^7}{7}\ln(x) - \frac{x^7}{49} + C}$$

Integration by Parts

10. Find the integral:

$$\int \pi e^{25} dx = \boxed{\pi e^{25} x + C}$$

11. Find the integral:

$$\int \frac{\sin^3(x)}{\cos(x)}dx = \int \frac{\sin^2(x)}{\cos(x)}\cdot\sin(x)dx = \int \frac{1-\cos^2(x)}{\cos(x)}\cdot\sin(x)\,dx$$

$u = \cos(x)$
$du = -\sin(x)dx$

$$= \int \frac{1-u^2}{u}(-du)$$
$$= -\int\left(\frac{1}{u} - u\right)du$$
$$= -\left(\ln|u| - u^2/2\right) + C$$
$$= \boxed{-\ln|\cos(x)| + \frac{\cos^2(x)}{2} + C}$$

12. Find the integral:

$$\int \arctan(x)dx = \int (x)'\arctan(x)dx = x\arctan(x) - \int x\,(\arctan'(x))dx$$
$$= x\arctan(x) - \int \frac{x}{1+x^2}dx$$

$u = 1 + x^2$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

$$= x\arctan(x) - \int \frac{1}{u}\cdot\frac{1}{2}du$$
$$= x\arctan(x) - \frac{1}{2}\ln|u| + C$$
$$= \boxed{x\arctan(x) - \frac{1}{2}\ln(1+x^2) + C}$$

13. Find the integral:

$$\int \frac{1}{2\sqrt{x+1}+x\sqrt{x+1}}dx = \int \frac{1}{\sqrt{x+1}\,(x+2)}\,dx = \int \frac{1}{\sqrt{x+1}\,\left(1+(\sqrt{x+1})^2\right)}\,dx$$

$$u = \sqrt{x+1}$$
$$du = \frac{1}{2\sqrt{x+1}}dx$$
$$2du = \frac{1}{\sqrt{x+1}}dx$$

$$= \int \frac{1}{1+u^2}\,2du$$
$$= 2\arctan(u) + C$$
$$= \boxed{2\arctan(\sqrt{x+1}) + C}$$

14. Find the integral:

$$\int \cos(\sqrt{x})dx$$

$$u = \sqrt{x}$$
$$u^2 = x$$
$$2u\,du = dx$$

$$\int \cos(\sqrt{x})dx = \int \cos(u)\cdot 2u\,du = 2\int u\cos(u)du$$
$$= 2\int u(\sin(u))'du$$
$$= 2\left[u\sin(u) - \int u'\sin(u)du\right]$$
$$= 2\left[u\sin(u) - \int \sin(u)du\right]$$
$$= 2\left[u\sin(u) + \cos(u) + C\right]$$
$$= \boxed{2\sqrt{x}\sin(\sqrt{x}) + 2\cos(\sqrt{x}) + C}$$