Worksheet (2) – Solutions

(1) a). Vector c). Scalar e). Vector

b). Scalar d). Scalar

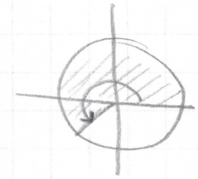
(2) $\|\vec{u}\| = \sqrt{2}$; $\|\vec{v}\| = 1.$

a). $\vec{u}\cdot\vec{v} = \|\vec{u}\|\,\|\vec{v}\| \cos(\pi/4)$

$= \sqrt{2}\cdot\frac{1}{\sqrt{2}} = \boxed{1}$

b). $\vec{u}\cdot\vec{v} = \|\vec{u}\|\,\|\vec{v}\| \cos(3\pi/4)$

$= \sqrt{2}\cdot\frac{-1}{\sqrt{2}} = \boxed{-1}$



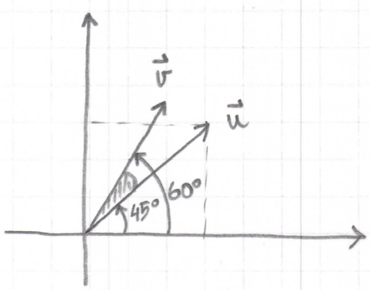
(3) $\|\vec{u}+\vec{v}\|^2 = (\vec{u}+\vec{v})\cdot(\vec{u}\cdot\vec{v})$

$= \vec{u}\cdot\vec{u} + \vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{u} + \vec{v}\cdot\vec{v}$

$= \|\vec{u}\|^2 + 2\,\vec{u}\cdot\vec{v} + \|\vec{v}\|^2$

So if $\|\vec{u}+\vec{v}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2$, then $2\,\vec{u}\cdot\vec{v} = 0$, thus $\vec{u}\perp\vec{v}$.

(4) $\|\vec{u}\| = \|\vec{v}\| = 1.$

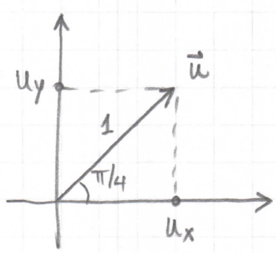


The angle between $\vec{u}$ and $\vec{v}$ is $15°$, so

$$\cos(15°) = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \vec{u}\cdot\vec{v}$$

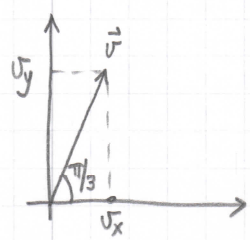
$$\left.\begin{array}{l}\vec{u} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle \\ \vec{v} = \left\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle\end{array}\right\} \Rightarrow \cos(15°) = \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$$

$$\boxed{\cos(15°) = \frac{1+\sqrt{3}}{2\sqrt{2}}}$$



$\cos\frac{\pi}{4} = \frac{u_x}{1} \Rightarrow u_x = \frac{1}{\sqrt{2}}$

$\sin\frac{\pi}{4} = \frac{u_y}{1} \Rightarrow u_y = \frac{1}{\sqrt{2}}$



$\cos\frac{\pi}{3} = \frac{v_x}{1} \Rightarrow v_x = \frac{1}{2}$

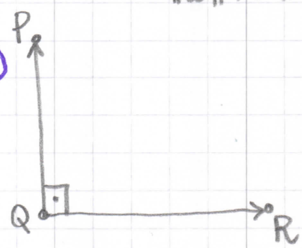
$\sin\frac{\pi}{3} = \frac{v_y}{1} \Rightarrow v_y = \frac{\sqrt{3}}{2}$

(5) $\vec{b} = \langle 2, 2, 1 \rangle$ $\qquad \|\vec{b}\| = 3$

$\theta = \frac{\pi}{6}$; $\vec{a} \cdot \vec{b} = 6$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \|\vec{b}\|} \Rightarrow \frac{\sqrt{3}}{2} = \frac{6}{\|\vec{a}\| \cdot 3} \Rightarrow \boxed{\|\vec{a}\| = \frac{4}{\sqrt{3}}}$$

(6)



$P(a, 1, -1)$

$Q(0, 1, 1)$

$R(a, -1, 3)$

$\vec{QP} = \langle a, 0, -2 \rangle$

$\vec{QR} = \langle a, -2, 2 \rangle$

$\vec{QP} \cdot \vec{QR} = 0$

$a^2 - 4 = 0 \Rightarrow \boxed{a = \pm 2}$

(7) a). $\vec{u} \cdot \vec{v} = \cos^2\theta + \sin^2\theta - 1 = 1 - 1 = 0 \Rightarrow$ <u>orthogonal</u>

b). $\vec{u} \cdot \vec{v} = \cos\theta \sin\theta - \sin\theta\cos\theta - 1 = -1 \Rightarrow$ <u>not orthogonal</u>

(8) $|\vec{u} \cdot \vec{v}| = \big| \|\vec{u}\| \|\vec{v}\| \cos\theta \big| = \|\vec{u}\| \|\vec{v}\| \, |\cos\theta| \leq \|\vec{u}\| \|\vec{v}\|$, because $|\cos\theta| \leq 1$.

(9) $\|\vec{u} + \vec{v}\|^2 = (\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v}) = \vec{u} \cdot \vec{u} + \vec{u} \cdot \vec{v} + \vec{v} \cdot \vec{u} + \vec{v} \cdot \vec{v}$

$= \|\vec{u}\|^2 + 2\, \vec{u} \cdot \vec{v} + \|\vec{v}\|^2$

$\|\vec{u} - \vec{v}\|^2 = (\vec{u} - \vec{v}) \cdot (\vec{u} - \vec{v}) = \vec{u} \cdot \vec{u} - \vec{u} \cdot \vec{v} - \vec{v} \cdot \vec{u} + \vec{v} \cdot \vec{v}$

$= \|\vec{u}\|^2 - 2\, \vec{u} \cdot \vec{v} + \|\vec{v}\|^2$

$\Rightarrow \|\vec{u} + \vec{v}\|^2 + \|\vec{u} - \vec{v}\|^2 = 2\|\vec{u}\|^2 + 2\|\vec{v}\|^2.$