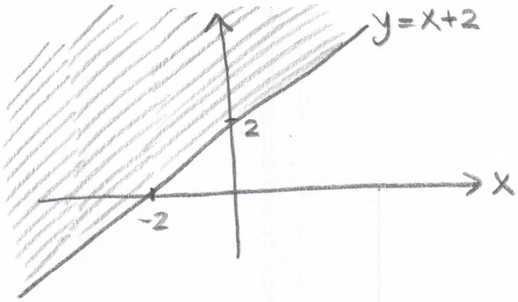
Worksheet 6

1 & 2 a) $f(x,y) = \sqrt{y-x-2}$

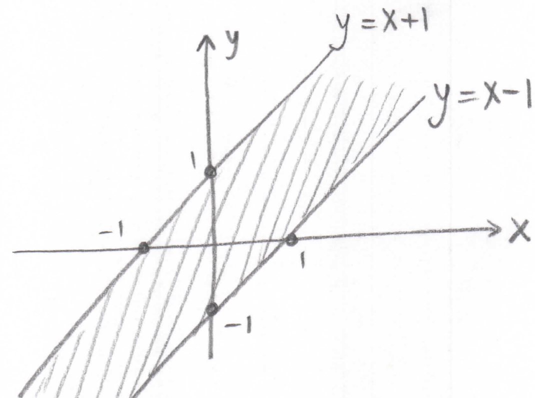


Domain: (x,y) such that $y-x-2 \geq 0$;
$y \geq x+2$

Range: $y-x-2 \geq 0 \Rightarrow \sqrt{y-x-2} \geq 0$

$\boxed{z \geq 0 \text{ or } [0,\infty)}$

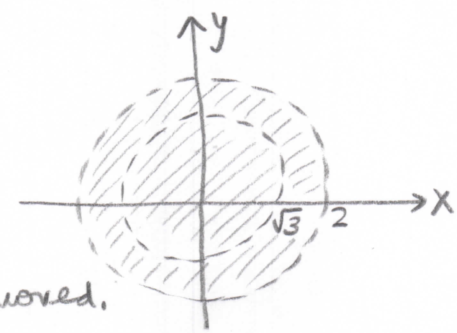
b) $f(x,y) = \arcsin(y-x)$



Domain: (x,y) s.t. $-1 \leq y-x \leq 1$
$x-1 \leq y \leq x+1$

Range: $-1 \leq y-x \leq 1$
$\Rightarrow -\frac{\pi}{2} \leq \arcsin(y-x) \leq \frac{\pi}{2}$
$\Rightarrow \boxed{[-\pi/2, \pi/2]}$

c) $f(x,y) = \dfrac{1}{\ln(4-x^2-y^2)}$



Domain: (x,y) s.t. $\begin{cases} 4-x^2-y^2 > 0 \\ 4-x^2-y^2 \neq 1 \end{cases}$ $\quad \begin{cases} x^2+y^2 < 4 \\ x^2+y^2 \neq 3 \end{cases}$

Interior of disk $x^2+y^2 \leq 4$, with the circle $x^2+y^2=3$ removed.

Range: $\begin{cases} x^2+y^2 < 4 \\ x^2+y^2 \neq 3 \end{cases}$ $\Rightarrow$ the domain splits into two regions, which must be analyzed separately:

I $\quad 0 \leq x^2+y^2 < 3$
$0 \geq -x^2-y^2 > -3$
$4 \geq 4-x^2-y^2 > 1$
$\ln(4) \geq \ln(4-x^2-y^2) > 0$
$\boxed{\dfrac{1}{\ln(4)} \leq \dfrac{1}{\ln(4-x^2-y^2)} < \infty}$

II $\quad 3 < x^2+y^2 < 4$
$-3 > -x^2-y^2 > -4$
$1 > 4-x^2-y^2 > 0$
$0 > \ln(4-x^2-y^2) > -\infty$
$\boxed{-\infty < \dfrac{1}{\ln(4-x^2-y^2)} < 0}$

Range: $\boxed{(-\infty, 0) \cup \left[\dfrac{1}{\ln(4)}, \infty\right)}$

③ a). $f(x,y) = \dfrac{xy}{x^6+y^2}$

$$\left. f(x,y) \right|_{y=x} = \frac{x^2}{x^6+x^2} = \frac{1}{x^4+1} \xrightarrow[x \to 0]{} 1$$

$$\left. f(x,y) \right|_{y=2x} = \frac{x \cdot 2x}{x^6+4x^2} = \frac{2}{x^4+4} \xrightarrow[x \to 0]{} \frac{1}{2}$$

} Limit DNE by 2-path test.

b). $\left. f(x,y) \right|_{y=kx} = \dfrac{x \cdot kx}{x^6+k^2x^2} = \dfrac{k}{x^4+k^2} \xrightarrow[x \to 0]{} \dfrac{1}{k}$ (dependence on k)

$\Rightarrow$ Limit DNE by 2-path test.

④ $f(x,y) = \dfrac{x^3 y}{x^6+y^2}$

$$\left. f(x,y) \right|_{y=kx} = \frac{x^3 kx}{x^6+k^2x^2} = \frac{kx^2}{x^4+k^2} \xrightarrow[x \to 0]{} 0$$

$$\left. f(x,y) \right|_{y=kx^2} = \frac{x^3 kx^2}{x^6+k^2x^4} = \frac{kx}{x^2+k^2} \xrightarrow[x \to 0]{} 0$$

$$\left. f(x,y) \right|_{y=kx^3} = \frac{x^3 kx^3}{x^6+k^2x^6} = \frac{k}{1+k^2}$$ dependence on k

$\Rightarrow$ Limit DNE by 2-path test.