Math 2401 Name:______________________________
Final Exam Section:______________________________

I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech community. By signing my name below I pledge that I have neither given nor received help on this exam.

Pledged:______________________________

Problem	Possible	Earned
1	10	
2	5	
3	5	
4	10	
5	10	
6	5	
7	10	
8	5	
9	10	
10	10	
11	5	
12	10	
13	10	
14	10	
15	10	
Total	125	

1. (10 pts) Let $-1 \leq t \leq 1$. For the parametric curve:

$$\vec{r}\,(t) = \left(\frac{4}{9}\,(1+t)^{\frac{3}{2}}\,,\, \frac{4}{9}\,(1-t)^{\frac{3}{2}}\,,\, \frac{t}{3} \right)$$

(a) Find the unit tangent vector to the curve $\vec{r}\,(t)$;

(b) Find the unit normal vector to the curve $\vec{r}\,(t)$;

(c) Find the unit binormal vector to the curve $\vec{r}\,(t)$.

2. (5 pts) Find the length of the curve

$$\vec{r}\,(t) = \left(3\cos t, 3\sin t, 2t^{\frac{3}{2}}\right)$$

when $0 \leq t \leq 3$.

3. (5 pts) Find an equation for the plane that passes through the point $(3, -2, 1)$ that is normal to the vector $\vec{n} = (2, 1, 1)$.

4. (10 pts) Find the directional derivative of

$$f(x, y, z) = \ln\left(x^2 + y^2 + z^2\right)$$

at the point (a, a, a), $a \neq 0$, in the direction of the vector $(2, 1, 2)$.

5. (10 pts) Find the maximum and minimum values of

$$f(x, y, z) = x + y - z$$

subject to the constraint that

$$\frac{x^2}{4} + \frac{y^2}{4} + z^2 = 1.$$

6. (5 pts) Find the limit of

$$f(x,y) = \frac{x^2 y}{x^4 + y^2}$$

as $(x,y) \to (0,0)$ or show that the limit does not exist.

7. (10 pts) For the function:

$$f(x, y) = -\frac{1}{4}x^4 + \frac{2}{3}x^3 + 4xy - y^2$$

(a) Find the critical values;

(b) For each critical point found from part (a) determine if the function achieves a local maximum, local minimum or a saddle point by using the second derivative test.

8. (5 pts) For the equation $x^2 + y^2 - 2xy - x + 3y - z = -4$ find the normal line through the point $(2, -3, 18)$.

9. (10 pts) Let Ω be the region in the xy plane that lies in the first quadrant, is bounded by the hyperbola $xy = 1$, $xy = 16$ and the lines $y = x$ and $y = 9x$.

(a) Sketch the region Ω;

(b) Find a domain Γ in the uv-plane so that the transformation $x(u, v) = \frac{u}{v}$ and $y(u, v) = uv$ with $u > 0$ and $v > 0$ maps Γ to Ω. Sketch Γ;

(c) Find the Jacobian of the above transformation;

(d) Evaluate $\iint_\Omega \left(\sqrt{\frac{y}{x}} + \sqrt{xy}\right)\, dA(x, y)$.

10. (10 pts) Find the volume of the region in the first octant bounded by the coordinate planes and the surface $z = 4 - x^2 - y$.

11. (5 pts) Use polar coordinates to evaluate the integral:

$$\int_{-1}^{1}\left(\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{\left(1+x^2+y^2\right)^2}\,dy\right)dx.$$

12. (10 pts) For the vector field $\vec{F}(x, y, z) = (e^x \cos y + yz, xz - e^x \sin y, xy + z)$:

(a) Show that $\vec{F}$ is a conservative vector field;

(b) Find the scalar potential function f such that $\vec{F} = \nabla f$;

(c) Evaluate the line integral of $\vec{F}$ over any path $\vec{r}$ starting at the point $(0, 0, 0)$ and ending at the point $(1, 2\pi, 0)$. Namely, compute

$$\int_C \vec{F} \cdot d\vec{r}$$

where C is any path starting at $(0, 0, 0)$ and ending at $(1, 2\pi, 0)$.

13. (10 pts) Let S denote the surface $x^2 + y^2 + z^2 = R^2$, and let $\vec{n}\,(x, y, z)$ denote the outer unit normal to the surface. Suppose that

$$\vec{v}\,(x, y, z) = \left(x\sqrt{x^2 + y^2 + z^2}, y\sqrt{x^2 + y^2 + z^2}, z\sqrt{x^2 + y^2 + z^2} \right).$$

Compute $\iint_S (\vec{v} \cdot \vec{n})\ d\sigma$ in two different ways:

(a) By direct calculation of $\iint_S (\vec{v} \cdot \vec{n})\ d\sigma$;

(b) By the Divergence Theorem.

You may use the fact that the surface area of the sphere of radius R is $4\pi R^2$, namely:

$$\iint_S d\sigma = 4\pi R^2.$$

14. (10 pts) Let S be the surface parameterized by $\vec{r}(u,v) = (u+v, u-v, v)$ with $0 \le u \le 1$ and $0 \le v \le 1$. Let

$$f(x,y,z) = xy - z^2.$$

Compute

$$\iint_S f \, d\sigma.$$

15. (10 pts) Let $\Omega = \{(x, y) : x^2 + y^2 \leq 1\}$. Let C be the boundary of the domain Ω, parameterized by the curve $\vec{r}\,(t) = (\cos t, \sin t)$ with $0 \leq t \leq 2\pi$. Show that

$$\oint_C ky\,dx + hx\,dy = (h - k)\pi$$

in two ways:

(a) By direct calculation of $\oint_C ky\,dx + hx\,dy$;

(b) By an application of Green's Theorem.