A New Approach to the Modeling and Analysis of Fracture through Extension of Continuum Mechanics to the Nanoscale

Tsvetanka Sendova
Institute of Mathematics and Its Applications
University of Minnesota
Minneapolis, MN 55403
sendova@ima.umn.edu

Jay R. Walton*
Department of Mathematics
Texas A&M University
College Station, TX 77843-3368
jwalton@math.tamu.edu

August 26, 2008

Abstract

This paper focuses on the analysis of the partial differential equations arising from a new approach to modeling brittle fracture based on an extension of continuum mechanics to the nanoscale. It is shown that ascribing constant surface tension to the fracture surfaces and using the appropriate crack surface boundary condition given by the jump momentum balance, leads to a sharp crack opening profile at the crack tip but predicts logarithmically singular crack tip stress. However a modified model, where the surface excess property is responsive to the curvature of the fracture surfaces, yields bounded stresses and a cusp-like opening profile at the crack tip. Further, two possible fracture criteria in the context of the new theory are discussed. The first one is an energy based crack growth condition, while the second employs the finite crack tip stress the model predicts. The classical notion of energy release rate is based upon the singular solution, whereas for the modeling approach adopted here, a notion analogous to the energy release rate arises through a different mechanism associated to the rate of working of the surface excess properties at the crack tip.

*Corresponding author.