HW 03. 2

Solve for \( x \). Give an exact answer: an integer, a fraction, or a decimal that terminates (i.e. \( 3/8 = 0.375 \)). If an answer does not exist, enter DNE.

\[
\frac{1}{2^x} = 2^{x-7}
\]

\[
\Rightarrow 2^{-7x} = 2^{x-7}
\]

\[
\Rightarrow -7x = x - 7
\]

\[
\Rightarrow -9x = -7
\]

\[
\therefore x = \frac{7}{9}
\]
HW #4, 15

How long will it take a $500 investment to be worth $600 if it is continuously compounded at 11% per year? (Give your answer to three decimal places.)

\[ A = P \cdot e^{rt} \]

\[ 600 = 500 \cdot e^{0.11t} \]

\[ \log(1.2) = \log(e^{0.11t}) \]

\[ \ln \frac{6}{5} = 0.11t \]

\[ t = \frac{\ln \frac{6}{5}}{0.11} \approx 1.657 \text{ yrs} \]
Solve the following equation for $x$. Round your answer to three decimal places.

$$\log_2(4x - 12) + \log_2(x + 5) = 2\log_2(2x)$$

$x =$ 7.500

\[\Rightarrow \log_2((4x - 12)(x + 5)) = \log_2(2x)^2\]

\[\Rightarrow (4x - 12)(x + 5) = (2x)^2\]

\[\Rightarrow 4x^2 + 8x - 60 = 4x^2\]

\[\Rightarrow 8x = 60\]

\[\therefore x = \frac{60}{8} = \frac{15}{2}\]
Find the derivative of the function.

\[ g(x) = \frac{e^x}{(x+\eta)^6} \]

\[ F = e^x \]
\[ F' = e^x \]
\[ G = (x^4 \eta)^4 \]
\[ G' = 4(x^4 \eta^3) \cdot (7x^2) \]
\[ = 12x^2 (x^4 \eta)^3 \]

\[ j'(x) = \frac{F' \cdot G - F \cdot G'}{G^6} \]
\[ = \frac{e^x \cdot (x^4 \eta)^3 - e^x \cdot 12x^2 \cdot (x^4 \eta)^3}{(x^4 \eta)^6} \]
Find the following derivative.

\[
\frac{d}{dx} \left( \frac{(10 - x^2)^3}{x^5} \right) = \frac{-60x^4 - 5(10 - x^2)^2(2x^3 + 10)}{x^6}
\]

\[ F = (10 - x^2)^3 \quad G = x^5 \]
\[ F' = 3(10 - x^2)^2 (-2x) \quad G' = 5x^4 \]
\[ = -15x^4 (10 - x^2)^2 \]

\[
\left( \frac{(10 - x^2)^3}{x^5} \right)' = \frac{-15x^4 (10 - x^2)^2 \cdot x^5 - (10 - x^2)^3 \cdot 5x^4}{x^{10}}
\]
Differentiate the function.

\[ y = (\ln(1 + e^x))^5 \]

\[ y' = \frac{6e^x (\ln(1 + e^x))^5}{1 + e^x} \]

\[ y' = 6 (\ln(1 + e^x))^4 \cdot \frac{1}{1 + e^x} \cdot e^x \]

\[ = \frac{6e^x (\ln(1 + e^x))^4}{1 + e^x} \]
Differentiate these functions.

\[ f(x) = \log_6(7 - x^5) \]

\[ f'(x) = \frac{-5x^4}{(7 - x^5) \cdot \ln(6)} \]

\[ g(x) = 5^{x^{11} + x} \]

\[ g'(x) = 5^{x^{11} + x} \cdot \ln5 \cdot (11x^{10} + 1) \]
A fast-food outlet finds that the demand equation for its new side dish is given by

\[ p = D(x) = \frac{128}{(x + 1)^2} \]

where \( p \) is the price (in cents) per serving and \( x \) is the number of servings that can be sold per hour at this price. At this time, the franchise is prepared to supply \( x \) servings per hour at a price of \( p \) cents based on the supply function given by

\[ p = S(x) = 2x + 2 \]

Find the equilibrium price, the consumers’ surplus \( CS \) and the producers’ surplus \( PS \) at this price level.

\[
\begin{align*}
\text{equilibrium price} &\quad \rho_0 = 8 \quad x_0 = 3, \quad \rho_0 = 8 \\
\text{consumers' surplus} &\quad CS = 72 \\
\text{producers' surplus} &\quad PS = 9
\end{align*}
\]

\[
\begin{align*}
\text{consumers' surplus} &\quad CS = \int_{x_0}^{p_0} \left[ D(x) - \rho_0 \right] dx \\
\text{producers' surplus} &\quad PS = \int_{x_0}^{p_0} \left[ \rho_0 - S(x) \right] dx
\end{align*}
\]
The Physics Club sells $E = mc^2$ T-shirts at the local flea market. Unfortunately, the club’s previous administration has been losing money for years, so you decide to do an analysis of the sales. A quadratic regression based on old sales data reveals the following demand equation for the T-shirts:

$$x = -4p^2 + 42p \quad (6 \leq p \leq 9).$$

Here, $p$ is the price the club charges per T-shirt, and $x$ is the number of shirts it can sell each day at the flea market.

(a) Obtain a formula for the price elasticity of demand for $E = mc^2$ T-shirts.

$$E(p) = \frac{-p \cdot f'(p)}{f(p)} = \frac{-p \cdot (-8p + 41)}{-4p^2 + 42p} = \frac{8p^2 - 41p}{-4p^2 + 42p} = \frac{4p - 21}{-2p + 41}$$

(b) Compute the elasticity of demand if the price is set at $6.50 per shirt. (Round your answer to two decimal places.)

$$E(6.50) = \frac{4(6.50) - 21}{-2(6.50) + 41} = \frac{26 - 21}{-13 + 41} = \frac{5}{28} \approx -0.18 \% \text{ change in demand per } \% \text{ change in price}$$

Interpret the result:

The demand for $E = mc^2$ T-shirts is going down by about $0.18 \%$ per $1\%$ increase in the price.

(c) How much should the Physics Club charge for the T-shirts in order to obtain the maximum daily revenue?

$$E(p) = \frac{4p - 21}{-2p + 41} = 1$$

What will the maximum revenue be?

$$R = p \cdot x$$

$$= 7 \cdot 98$$

$$= 686$$
ABC Daycare wants to build a fence to enclose a rectangular playground. The area of the playground is 910 square feet. The fence along three of the sides costs $5 per foot and the fence along the fourth side, which will be made of brick, costs $15 per foot. Find the length of the brick fence that will minimize the cost of enclosing the playground. (Round your answer to one decimal place.)

\[ x \cdot y = 910 \]
\[ y = \frac{910}{x} \]

Cost: \[ C = 5 \left( x + 2y \right) + 15 \cdot x \]
\[ \Rightarrow C(x) = 5 \left( x + 2 \cdot \frac{910}{x} \right) + 15x \]
\[ = 5x + \frac{910}{x} + 15x \]
\[ = 20x + \frac{910}{x} \]
\[ \Rightarrow C'(x) = 20 - \frac{910}{x^2} \]
\[ \Rightarrow 20 - \frac{910}{x^2} = 0 \]
\[ \Rightarrow \frac{910}{x^2} = 20 \]
\[ \Rightarrow 2x^2 = 91 \]
\[ \Rightarrow x^2 = \frac{91}{2} \]
\[ \Rightarrow x = \sqrt{\frac{91}{2}} \]

**Second Derivative Test**

\[ C''(x) = 1800x^{-3} = \frac{1800}{x^3} \]

\[ C''\left(\sqrt{\frac{91}{2}}\right) = \frac{1800}{\left(\frac{91}{2}\right)^{3/2}} > 0 \]

\[ \Rightarrow \text{local min at } x = \sqrt{\frac{91}{2}} \]

\[ \Rightarrow \text{Abs min} \]

\[ \therefore \text{Brick: } \sqrt{\frac{91}{2}} \text{ ft.} \]
A rancher wants to create two rectangular pens, as shown in the figure, using an existing fence line as one side. If there are 696 feet of fence available, what dimensions should be used to maximize the total area of the pens?

\[ 3x + y = 696 \]
\[ \Rightarrow y = 696 - 3x \]

Area: \[ A = x \cdot y \]
\[ \Rightarrow A(x) = x \cdot (696 - 3x) \]
\[ = 696x - 3x^2 \]

\[ \Rightarrow A(x) = -3x^2 + 696x \]
\[ \text{vertex: } x = \frac{-b}{2a} = \frac{-696}{2 \cdot (-3)} = 116 \]

\[ y = 696 - 3(116) = 348 \]

\[ 116 \times 348 \]
If 30,000 cm$^3$ of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

Material: $A = x^2 + 4xy = 30000$

Volume: $V = x^2y$

$\Rightarrow \Delta x y = 30000 - x^2$

$\Rightarrow y = \frac{30000 - x^2}{4x}$

$\Rightarrow V(x) = x^2 \left( \frac{30000 - x^2}{4x} \right)$

$\Rightarrow V(x) = \frac{30000}{4} x - \frac{1}{4} x^5$

$\Rightarrow V'(x) = 7500 - \frac{3}{4} x^4 = 0$

$\Rightarrow 2500 = \frac{3}{4} x^4 \Rightarrow x^4 = \frac{10000}{3}$

$\Rightarrow x = \sqrt[4]{\frac{10000}{3}}$

$\Rightarrow x = 100$

Second Derivative Test

$V''(x) = -\frac{3}{4}x^2$

$\Rightarrow V''(100) = -\frac{3}{4}(100)^2 < 0$

$\therefore$ local Max at $x = 100$

$y = \frac{30000 - (100)^2}{4(100)}$

$= \frac{30000 - 10000}{400}$

$= \frac{20000}{400} = 50$

$\therefore$ Max Volume $= 100 \times 100 \times 50$

$= 500000$ cm$^3$
Macrossoft produces two versions of its popular gaming console: the Elite and the Casual. The weekly demand and cost functions for the consoles are

\[
\begin{align*}
  p &= 300 - 8x + 2y \\
  q &= 225 - x + 6y \\
  C(x, y) &= 400 + 90x + 120y
\end{align*}
\]

where \( x \) represents the weekly demand for the Elite version; \( y \) represents the weekly demand for the Casual version; \( p \) and \( q \) represent the price (in dollars) of an Elite console and a Casual console, respectively; and \( C(x, y) \) is the cost function.

(a) Determine \( R(x, y) \), the weekly revenue function.
\[
R(x, y) = px + qy = (300 - 8y + 2y) - x + (225 - x + 6y) y.
\]

(b) Determine \( P(x, y) \), the weekly profit function.
\[
P(x, y) = R(x, y) - C(x, y)
\]

(c) Find \( P(8, 2) \).
\[
P(8, 2) = \left[ (300 - 8x + 2y) - x + (225 - x + 6y) y \right] - (400 + 90x + 120y) \]
A manufacturer has modeled its yearly production function $P$ (the monetary value of its entire production in millions of dollars) as a Cobb-Douglas function

$$P(L, K) = 1.47L^{0.65}K^{0.35}$$

where $L$ is the number of labor hours (in thousands) and $K$ is the invested capital (in millions of dollars). Find $P(135, 25)$ and interpret it. (Round your answers to one decimal place.)

$$P(135, 25) = [\text{No Response}]$$

so when the manufacturer invests $\$[\text{No Response}]$ million in capital and $[\text{No Response}]$ thousand hours of labor are completed yearly, the monetary value of the production is about $\$[\text{No Response}]$ million.
Homework 8

If $f(4) = 10$, $f'$ is continuous, and $\int_4^7 f'(x) \, dx = 16$, what is the value of $f(7)$?

$f(7) = \boxed{\text{(No Response)}}$

\[
\begin{align*}
\text{T.T. C. 2} & \\
\int_4^7 f'(x) \, dx &= f(x) \bigg|_4^7 = f(7) - f(4) \\
\Rightarrow f(7) - f(4) &= 16 \\
\Rightarrow f(7) - 10 &= 16 \\
\therefore f(7) &= 16 + 10 = 26
\end{align*}
\]
Suppose that copper is being projected to be extracted from a certain mine at a rate given by

\[ P(t) = 200e^{-0.08t} \]

where \( P(t) \) is measured in tons of copper and \( t \) is measured in years.

(a) How many tons of copper is projected to be extracted during the second five year period? Give you answer to three decimal places.

\[
\int_5^9 P(t) \, dt
\]

(No Response) Tons

(b) How many tons of copper is projected to be extracted during the sixth and seventh years? Give you answer to three decimal places.

(No Response) Tons
A company sells gadgets and widgets. The gadgets sell at \( p = 1168 - 5x - 2y \) and the widgets sell at \( q = 912 - 4x - 3y \) where \( x \) is the number of gadgets sold, \( y \) is the number of widgets sold and \( p \) and \( q \) are in dollars.

(a) Find the company’s revenue function.
\[
R(x, y) = px + qy = (1168 - 5x - 2y)x + (912 - 4x - 3y)y
\]

(b) Find the number of gadgets and widgets the company must sell to obtain maximum revenue.

Find C.P.
\[
\begin{align*}
R_x &= -10x + 1168 - 6y + 0 = 0 \\
\Rightarrow -10x - 6y + 1168 &= 0 \\
\Rightarrow 10x + 6y &= 1168 \\
\Rightarrow 5x + 3y &= 584 \\
R_y &= 0 + 0 - 6x + 912 - 4y = 0 \\
\Rightarrow -6x - 4y + 912 &= 0 \\
\Rightarrow 6x + 4y &= 912 \\
\Rightarrow x + y &= 152 \\
\end{align*}
\]
\[
\begin{align*}
5x + 3y &= 584 \\
x - y &= 152 \\ \\
5x + 3(152 - x) &= 584 \\
5x + 456 - 3x &= 584 \\
2x &= 128 \\
\Rightarrow x &= 64 \\
y &= 152 - 64 = 88 \\
\end{align*}
\]
\[\therefore \text{ C.P.: } (64, 88)\]

\[
D(x,y) = R_{xx} \cdot R_{yy} - (R_{xy})^2
\]
\[
\begin{align*}
R_x &= -10x - 6y + 1168 \\
R_{xx} &= -10 \\
R_y &= -6x - 4y + 912 \\
R_{yy} &= -4 \\
R_{xy} &= -6
\end{align*}
\]
\[
D(x,y) = R_{xx} \cdot R_{yy} - (R_{xy})^2 = (-10)(-6) - (-6)^2 = 60 - 36 = 24
\]
\[
\Rightarrow D(64, 88) = 24 > 0
\]
\[
\Rightarrow \text{ local max at } (64, 88)
\]