We will learn some tests to deal with alternating series.

Alternating Series: An alternating series is a series where terms are alternating in sign (opposite). The series, or its alternating form $\sum_{n=1}^{\infty} a_n$ can be written as $\sum_{n=1}^{\infty} (-1)^n b_n$ or $\sum_{n=1}^{\infty} (-1)^{n+1} b_n$ where b_n is a positive number. $b_n \geq 0$ $\forall n \geq 1$

ex: $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

$\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \cdots = \sum_{n=2}^{\infty} \frac{(-1)^n}{n}$

$\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+1}$

$\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \cdots = \sum_{n=3}^{\infty} \frac{(-1)^n}{n-1}$

$\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \cdots = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n+2}$

$\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+1}$

Alternating form

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = \frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots$

$= \frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots$

Not alternating series

Alternating Series Test: If the alternating series

$\sum_{n=1}^{\infty} (-1)^n b_n$ or $\sum_{n=1}^{\infty} (-1)^{n+1} b_n$ $\quad b_n \geq 0$

satisfies

$\lim_{n\to\infty} b_n = 0$ $\quad$ decreasing

$b_{n+1} \leq b_n$ $\quad$ $\forall n \geq N$ for some $N \in \mathbb{N}$

then the series is convergent

Note: alternating series test REQUIRES one for the series.

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ $\quad b_n = \frac{1}{n}$

$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{n} = 0$

$b_{n+1} \leq b_n$ $\quad \forall n \geq N$

Alternating series test

$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{n} = 0$

$b_{n+1} \leq b_n$ $\quad \forall n \geq N$

$\frac{1}{n+1} \leq \frac{1}{n}$ $\quad \forall n \geq 1$

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ converges by the A.S.T

And $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ Alternating Harmonic Series

Convergent

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n+1}$ $\quad b_n = \frac{1}{n+1}$

$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{n+1} = 0$

$b_{n+1} \leq b_n$ $\quad \forall n \geq N$

Alternating series test

$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{n+1} = 0$

$b_{n+1} \leq b_n$ $\quad \forall n \geq N$

$\frac{1}{n+2} \leq \frac{1}{n+1}$ $\quad \forall n \geq 1$

Alternating form This Series!!

Use the Test for Divergence.

$\lim_{n\to\infty} (-1)^n \frac{n}{n+1} = \lim_{n\to\infty} \frac{n}{n+1}$ DNE

$\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+1}$ diverges by the Test for Divergence.

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1}$ $\quad b_n = \frac{1}{n^2+1}$

$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{n^2+1} = 0$

$b_{n+1} \leq b_n$ $\quad \forall n \geq N$

Alternating series test

$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{n^2+1} = 0$

$b_{n+1} \leq b_n$ $\quad \forall n \geq N$

$\frac{1}{(n+1)^2+1} \leq \frac{1}{n^2+1}$ $\quad \forall n \geq 1$

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1}$ converges by A.S.T

Note: Proof of an alternating series $\sum_{n=1}^{\infty} (-1)^n b_n$ lies the condition of the alternating series test.

Proof of an alternating series $\sum_{n=1}^{\infty} (-1)^n b_n$ lies the limit zero and, one, or two monotonic sequence, and the series $\sum_{n=1}^{\infty} (-1)^n b_n$ is the limit zero and, one, or two $\sum_{n=1}^{\infty} (-1)^n b_n$ lies the limit zero and, one, or two monotonic sequence.

Note: $\sum_{n=1}^{\infty} (-1)^n b_n$ lies the limit zero and, one, or two monotonic sequence, and the series $\sum_{n=1}^{\infty} (-1)^n b_n$ is the limit zero and, one, or two monotonic sequence.

Note: The end does not apply to other types of series.

Assume $\lim_{n\to\infty} b_n = 0$, then

$\sum_{n=1}^{\infty} (-1)^n b_n$ $\quad \sum_{n=1}^{\infty} (-1)^{n+1} b_n$

$\lim_{n\to\infty} b_n = 0$ $\quad$ Alternating series test

$\lim_{n\to\infty} b_n = 0$ $\quad$ Alternating series test

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$\lim_{n\to\infty} b_n = 0$ $\quad$ Alternating series test

Alternating Series Estimation Theorem: If $s = \sum_{n=1}^{\infty} (-1)^n b_n$ with $a_n \geq 0$ satisfies the alternating series test then

$|s - s_n| \leq b_{n+1}$

Note: The end does not apply to other types of series.

Assume $s = \sum_{n=1}^{\infty} (-1)^n b_n$ satisfies the alternating series test then

$|s - s_n| \leq b_{n+1}$

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