Marginal-[Cost, Revenue, and Profit]   Average Cost Average Revenue and Average Profit functions
Marginal Average Cost   Marginal Average Revenue and Marginal Average Profit

Introduction:

Recall the limit definition of the derivative $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$

If this limit exists and *h* is "small enough" then

$\frac{f(x+h) - f(x)}{h} \approx f'(x) \qquad f(x+h) - f(x) \approx h\, f'(x)$

***f(x+h)-f(x)* is the exact change in *f*   and   *hf* '(*x*) is the approximate change in *f*.**

If *x* is large, say in the thousands as in production quantities, then *h*=1 is relatively small. In that case $f(x+1) - f(x) \approx 1 f'(x) = f'(x)$. This is used to marginal cost, revenue and profit.

The marginal cost function is C '(x) , the marginal revenue function is R '(x), and the marginal profit function is P'(x).

Example: Given the profit function for producing and selling *x* units is

$P(x) = -.05x^2 + 150x - 1500$

a) Find the exact change in profit if the production level increases from 2000 to 2001.
b) Use the marginal profit function to approximate the change in profit if x increases from 2000 to 2001.

a) The exact change in profit is *P*(2001) - *P*(2000) = -$50.05. The profit decreases by $50.05. (We get this by substituting 2001 in to P and 2000 into P and subtracting).

b) To approximate this change, first find the marginal profit function, *P*'(*x*). Then substitute *x*=2000.

$P'(x) = -0.10x + 150 \qquad P'(2000) = -50$ which is only off by 5 cents.
This means that profit is decreasing by approximately $50 per additional unit when 2000 units are produced.

Average Cost, $\frac{C(x)}{x}$ **Average Revenue,** $\frac{R(x)}{x}$ **and Average Profit,** $\frac{P(x)}{x}$

$\frac{C(x)}{x}$ is total cost / #units = average cost per unit Similarly for the others**.**

Marginal Average Cost is $\frac{d}{dx}\left(\frac{C(x)}{x}\right)$ **= the derivative of the average cost function.**

Similarly for the others.

Example: Problem 10 pg 206 Section 3.7 in the text book by *Barnett, Ziegler and Byleen*

The total profit from the sale of x charcoal grills is $P(x) = -0.02x^2 + 20x - 320$

A. Find the average profit per grill if 40 grills are produced.

$$P(40)/40 = \frac{-0.02(40^2) + 20(40) - 320}{40} = 11.20$$

B. Find the marginal average profit at production level 40.
First find the average profit function, then take its derivative, then substitute x=40.

$$\frac{P(x)}{x} = \frac{-0.02x^2 + 20x - 320}{x} = -0.02x + 20 - 320x^{-1}$$

$$\frac{d}{dx}\left(\frac{P(x)}{x}\right) = -0.02 + 320x^{-2}$$

$$Plug\ \ in\ \ x = 40\ \ to\ \ get\ \ -0.02 + 320/1600 = -0.02 + 0.2 = 0.18$$

What does it mean? the average profit per grill is increasing by approximately 18 cents per additional grill when 40 grills are produced.

A common mistake is to find the average marginal profit. This is not used.
Always find the average (cost, revenue or profit) first and then take the derivative.
Remember: It's the marginal(average cost, average revenue or average profit)