

Notice that points that are outside of the feasible region have at least one slack variable that is negative.

The simplex method solves linear programming problems that can not be solved by the method of corners, i.e. graphing method. The simplex method only work with standard maximum linear programming problems.

Requirements for the simplex method.

- 1) Objective function is maximized.
- 2) All variables are non-negative.
- 3) All constraints are in the form:

$$ax + by + \dots \leq c \text{ with } c \geq 0$$

Examples of problems that will work with the simplex method:

Example 1:

$$\begin{aligned} \max f &= 2x + 5y \\ x + y &\leq 5 \\ 2x - 4y &\leq 2 \\ x, y &\geq 0 \end{aligned}$$

Example 2:

$$\begin{aligned} \max f &= 2x - 6y \\ x - 3y &\geq -4 \\ 3x + 5y &\leq 7 \\ x, y &\geq 0 \end{aligned}$$

Example 2 does not work as it is written. You would need to take the first constraint and multiply it by a -1 , $-x + 3y \leq 4$.

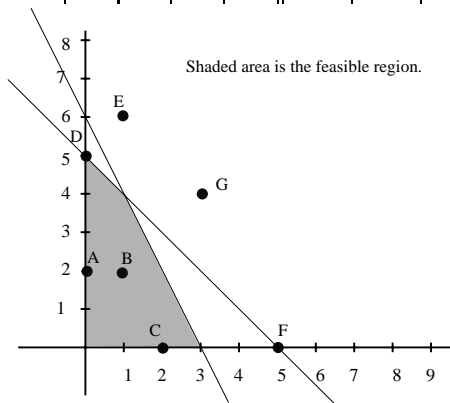
The different parts of the Simplex Method

Slack equations. The constraints are converted to equations by adding a different variable to each inequality, called a slack variable. The only constraints that are made into slack equations are the $x \geq 0, \dots$

Example:

Constraints:	Slack equations:
$x + y \leq 5$	$x + y + s_1 = 5$
$2x + y \leq 6$	$2x + y + s_2 = 6$
$x, y \geq 0$	

	A	B	C	D	E	F	G
x	0	1	2	0	5	1	3
y	2	2	0	5	0	6	4
s ₁	3	2	3	0	0	-2	-2
s ₂	4	2	2	1	-4	-2	-4



Basic and Non-Basic variables:

In a simplex matrix, the variables that have a unit column are called basic variables. A unit column is a column that is all zeros except for a single one. The variables that are not a unit column, i.e. a column of junk, are called non-basic variables.

Knowing these two definitions is not a important as being able to use this information when reading off the solution of a simplex matrix. The non-basic variables act like parameters and thus in the simplex process, we set their values to zero.

Example 1:

$$\left[\begin{array}{cccc|c} x & y & s_1 & s_2 & \\ 1 & 1 & 1 & 0 & 5 \\ 2 & 1 & 0 & 1 & 6 \end{array} \right]$$

In this matrix, s_1 and s_2 are the basic variables and x and y are the non basic variables. Thus we set both x and y to zero when we read off their solutions.

From the matrix we know that $x + y + s_1 = 5$ and $2x + y + s_2 = 6$. Since $x = 0$ and $y = 0$ this gives $0 + 0 + s_1 = 5$ and $2 * 0 + 0 + s_2 = 6$

Answer: $x = 0, y = 0, s_1 = 5, s_2 = 6$

Example 2:

$$\left[\begin{array}{cccc|c} x & y & s_1 & s_2 & \\ 1 & 1 & 1 & 0 & 5 \\ 1 & 0 & -1 & 1 & 1 \end{array} \right]$$

In this matrix, y and s_2 are the basic variables and x and s_1 are the non basic variables and thus are both set to zero.

Answer: $x = 0, y = 5, s_1 = 0, s_2 = 1$

During the simplex method, we have to find the pivot element of a simplex matrix. This is done by what is called the smallest quotient rule. To do this, we first need to know the pivot column of the matrix. I will explain how to find the pivot column in a little bit.

Smallest Quotient Rule:

1. Divide all **positive** numbers in the pivot column into the answer column (last column of the matrix).
2. The row that has the smallest non-negative quotient, i.e. is ≥ 0 , will be called the pivot row. If there is a tie for the smallest non-negative quotient then you can use either of the rows as the pivot row.
3. The number where the pivot row and pivot column intersect is the pivot element.

Example 1: Assume the first column is the pivot column.

$$\left[\begin{array}{ccccc|c} x & y & s_1 & s_2 & s_3 & \\ \hline 1 & 1 & 1 & 0 & 0 & 6 \\ -1 & 3 & 0 & 1 & 0 & 0 \\ 2 & -1 & 0 & 0 & 1 & 6 \end{array} \right] \quad \begin{array}{l} \text{Quotient} \\ 6/1 = 6 \\ - \\ 6/2 = 3 \end{array}$$

The third row is the pivot row and this gives the pivot element is in row 3 column 1.

Example 2: Assume the second column is the pivot column.

$$\left[\begin{array}{ccccc|c} x & y & s_1 & s_2 & s_3 & \\ \hline 1 & 1 & 1 & 0 & 0 & 6 \\ -1 & 3 & 0 & 1 & 0 & 0 \\ 2 & -1 & 0 & 0 & 1 & 6 \end{array} \right] \quad \begin{array}{l} \text{Quotient} \\ 6/1 = 6 \\ 0/3 = 0 \\ - \end{array}$$

The second row is the pivot row and this gives the pivot element is in row 2 column 2.

Setting up the Simplex Matrix

$$\begin{aligned} f &= 2x + 3y \text{ max} \\ x + y &\leq 8 \\ 3x + 5y &\leq 30 \\ x \geq 0, y &\geq 0 \end{aligned}$$

To set up the first simplex matrix for this linear programming problem, first write down the slack equations. $x + y + s_1 = 8$ and $3x + 5y + s_2 = 30$

Next, rewrite the objective function such that it is set equal to zero and the coefficient in front of f is a positive 1.

$$-2x - 3y + f = 0$$

now fill in the matrix.

$$\left[\begin{array}{ccccc|c} x & y & s_1 & s_2 & f & \\ \hline 1 & 1 & 1 & 0 & 0 & 8 \\ 3 & 5 & 0 & 1 & 0 & 30 \\ \hline -2 & -3 & 0 & 0 & 1 & 0 \end{array} \right]$$

Simplex Method.

1. Set up the initial simplex matrix.
2. Find the smallest negative number in the bottom row. This is the pivot column. If there is a tie, then it is your choice as to which column is the pivot column.
3. Do the smallest quotient rule to find the pivot element.
4. Pivot on the pivot element by either using the Gauss-Jordan row operations or the Simplex program on the calculator.
5. If there are still negative numbers in the bottom row, then go back to step 2 otherwise goto the nextg step.
6. Simplex process is done. read off the solution.

Solve this problem using the simplex method.

I'm using the simplex program to do the pivoting after i find the pivot element.

$$\begin{aligned} f &= 2x + 3y \text{ max} \\ x + y &\leq 8 \\ 3x + 5y &\leq 30 \\ x \geq 0, y &\geq 0 \end{aligned}$$

$$\left[\begin{array}{ccccc|c} x & y & s_1 & s_2 & f & \\ \hline 1 & 1 & 1 & 0 & 0 & 8 \\ 3 & 5 & 0 & 1 & 0 & 30 \\ \hline -2 & -3 & 0 & 0 & 1 & 0 \end{array} \right] \quad \begin{array}{l} \text{Quotient} \\ 8/1 = 8 \\ 30/5 = 6 \end{array} \quad \begin{array}{l} \text{P.C.} = \text{col 2} \\ \text{P.R.} = \text{row 2} \end{array}$$

$$\left[\begin{array}{ccccc|c} x & y & s_1 & s_2 & f & \\ \hline 2/5 & 0 & 1 & -1/5 & 0 & 2 \\ 3/5 & 1 & 0 & 1/5 & 0 & 6 \\ \hline -1/5 & 0 & 0 & 3/5 & 1 & 18 \end{array} \right] \quad \begin{array}{l} \text{Quotient} \\ 2/(2/5) = 5 \\ 6/(3/5) = 10 \end{array} \quad \begin{array}{l} \text{P.C.} = \text{col 1} \\ \text{P.R.} = \text{row 1} \end{array}$$

$$\left[\begin{array}{ccccc|c} x & y & s_1 & s_2 & f & \\ \hline 1 & 0 & 5/2 & -1/2 & 0 & 5 \\ 0 & 1 & -3/2 & 1/2 & 0 & 3 \\ \hline 0 & 0 & 1/2 & 1/2 & 1 & 19 \end{array} \right]$$

Answer: $x = 5, y = 3, s_1 = 0, s_2 = 0, f = 19$