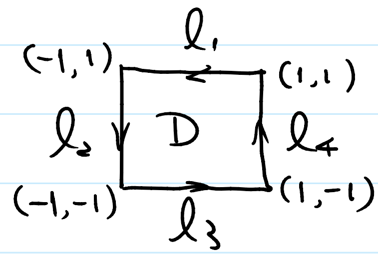
M311-HWK-10.2

Tuesday, April 21, 2020 9:37 AM

§10.2



#3 $F = y\,i + x^2 j$

$N_x - M_y = 2x - 1.$

$$RHS = \iint_D (\underbrace{2x}_{0 \text{ odd}} - 1)\,dx\,dy = -1\,|D| = -4$$

LHS: l_1: $x\big|_1^{-1}$, $y=1$, $dy=0$. l_2: $x=-1$, $dx=0$, $y\big|_1^{-1}$

l_3: $y=-1$, $dy=0$, $x\big|_{-1}^{1}$, l_4: $x=1$, $dx=0$, $y\big|_{-1}^{1}$.

$$LHS = \int_1^{-1} dx + \int_1^{-1} (-1)^2 dy + \int_{-1}^{1} (-1)\,dx + \int_{-1}^{1} 1^2 dy = -4$$

#5. $D: x^2 + 2y^2 \le 4 \iff \dfrac{x^2}{2^2} + \dfrac{y^2}{(\sqrt{2})^2} \le 1$.

$F = 3y\,i - 4x\,j$

LHS: $X(t) = (2\cos t, \sqrt{2}\sin t)$, $0 \le t \le 2\pi$.

$X'(t) = (-2\sin t, \sqrt{2}\cos t)$. $F = (3\sqrt{2}\sin t, -8\cos t)$

$$LHS = \int_0^{2\pi} (-6\sqrt{2}\sin^2 t - 8\sqrt{2}\cos^2 t)\,dt$$

$$= \int_0^{2\pi} \left(-3\sqrt{2}(1 - \underbrace{\cos 2t}_{0}) - 4\sqrt{2}(1 + \underbrace{\cos 2t}_{0})\right) dt$$

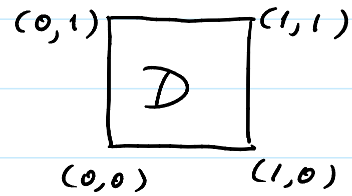
$$= -7\sqrt{2}\int_0^{2\pi} dt = -14\sqrt{2}\,\pi$$

RHS. $N_x - M_y = -4 - 3 = -7$

$$\iint_D (-7)\,dx\,dy = -7 \cdot \overbrace{2 \cdot \sqrt{2}\,\pi}^{(ab\pi)} = -14\sqrt{2}\,\pi.$$

#7.

$$\oint_C y^2dx + x^2dy = \iint_D (N_x - M_y)\,dx\,dy$$



$$= \iint_D (2x-2y)\,dx\,dy = \int_0^1\int_0^1 (2x-2y)\,dx\,dy = 0.$$

#16 Did in the lecture §10.2 P2.

§10.3

#1 $(0,0,0) \to (1,1,1)$

a) $x=y=z=t\big|_0^1$, $\int_C \underbrace{z^2dx+2ydy+xzdz}_{I} = \int_0^1 (t^2+2t+t^2)\,dt$

$= \frac{2}{3}+1$

b) $X(t)=(t,t^2,t^3)$ $t\big|_0^1$

$I=\int_0^1 (t^6+2t^2(2t)+t^4(3t^2))\,dt = 4/7+1.$

c) Not conservative

#9 $F=(6xy^2-3x^2,\ y^2+6x^2y)$, $M_y=12xy=N_x$, Yes

$f=\int M\,dx+\alpha(y) = 3x^2y^2-x^3+\alpha(y)$ $\quad \alpha(y)=\frac{1}{3}y^3$

$=\int N\,dy+\beta(x) = \frac{1}{3}y^3+3x^2y^2+\beta(x)$ $\gg$ $\beta(x)=-x^3$

#15 $F = (e^x \sin y,\ e^x \cos y,\ 3z^2+z)$.

$$\nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^x \sin y & e^x \cos y & 3z^2+2 \end{vmatrix} = 0 \quad \text{yes}$$

$$f = \int M\,dx + \alpha(y,z) = e^x \sin y + \alpha(y,z)$$
$$= \int N\,dy + \beta(x,z) = e^x \sin y + \beta(x,z) \Rightarrow \alpha(y,z) = \beta(x,z) = z^3 + 2z$$
$$= \int P\,dz + \gamma(x,y) = z^3 + 2z + \gamma(x,y) \quad \gamma(x,y) = e^x \sin y.$$

§ 11.1. $N = (a,b,c)$, T: $a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$.

#1. $X(s,t) = (s^2 - t^2,\ s+t,\ s^2+3t)$, $(3,1,1) = X(2,-1)$

$$N = T_s \times T_t \Big|_{(s,t)=(2,-1)} = (-1, -4, 2)$$

#11 $z = f(x,y)$, $N = (-f_x, -f_y, 1)$, $(x_0, y_0, z_0) = (1, 0, \sqrt{2})$

a) $f(x,y) = \sqrt{4 - (x-2)^2 - (y+1)^2}$,

$$f_x = {}^{-(x-2)}\!/z = {}^{1}\!/\sqrt{2}, \quad f_y = {}^{-(y+1)}\!/z = {}^{-1}\!/\sqrt{2}, \quad (x,y,z) = (1,0,\sqrt{2})$$

b) $F(x,y,z) = (x-2)^2 + (y+1)^2 + z^2 = 4 \Rightarrow z = z(x,y)$

$F_x = 2(x-2) + 2z\, z_x = 0 \Rightarrow z_x = {}^{-(x-2)}\!/z \Big|$

$$F_x = 2(x-2) + 2z\,z_x = 0 \Rightarrow z_x = \frac{-(x-2)}{z}\Bigg|_{\substack{(x,y,z)\\ =(1,0,\sqrt{2})}} = \frac{1}{\sqrt{2}}$$

$$F_y = 2(y+1) + 2z\,z_y = 0 \Rightarrow z_y = \frac{-(y+1)}{z} = \frac{-1}{\sqrt{2}}$$

or use N=(F_x, F_y, F_z)=(-2, 2, 2\sqrt(2))

c) $X(s,t) = (2\sin s\cos t + 2,\ 2\sin s\sin t - 1,\ 2\cos s)$

$(1, 0, \sqrt{2}) = X(s,t)$, Solve for s, t.

$\cos s = \frac{1}{\sqrt{2}} \Rightarrow s = \pi/4$, $\Rightarrow \sin s = \frac{1}{\sqrt{2}}$

then $2 \cdot \frac{1}{\sqrt{2}} \cos t + 2 = 1$, $\Rightarrow \cos t = -\frac{1}{\sqrt{2}}$, $\Rightarrow t = 3\pi/4$.

$$N = T_s \times T_t \Big|_{(s,t) = (\frac{\pi}{4}, 3\pi/4)} = (-\sqrt{2}, \sqrt{2}, 2)$$

§7.4, 8.4.

$u \times v$, $|u \times v| = \frac{1}{2}|\text{▱}|$, $u \times v \perp \begin{matrix} u \\ v \end{matrix}$

$F = (M, N, P)$. $\operatorname{div} = \nabla \cdot F = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$.

$$\operatorname{Curl} F = \nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix}$$

§10.1 #23

$(1, 1, 3)$ $(-1, 1, 3)$ $(-1, -1, 3)$ $(1, -1, 3)$,

$(1,1,3) \xrightarrow{l_1} (-1,1,3) \xrightarrow{l_2} (-1,-1,3) \xrightarrow{l_3} (1,-1,3) \xrightarrow{l_4}$

l_1: $x\big|_{1}^{-1}$, $y=1$, $z=3$. $dy=dz=0$

l_2: $x=-1$, $z=3$, $dx=dz=0$. $y\big|_{1}^{-1}$

l_3: $x\big|_{-1}^{1}$, $y=-1$, $z=3$, $dy=dz=0$

l_4: $x=1$, $z=3$, $y\big|_{-1}^{1}$, $dx=dz=0$

$\int M\,dx + N\,dy + P\,dz$.