HWK-11.2

Thursday, April 18, 2019 10:28 AM

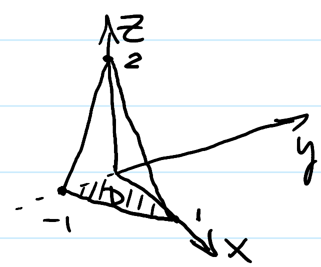
#1. Given $X(s,t) = (s, s+t, t)$, $0 \le s \le 1$, $0 \le t \le 2$

Find $\iint_X (x^2+y^2+z^2)\,dS = I$.

$$N = T_s \times T_t = \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = (1,-1,1) \Rightarrow \|N\| = \sqrt{3}$$

$$I = \int_0^2 \int_0^1 (s^2+(s+t)^2+t^2)\cdot\sqrt{3}\,ds\,dt = \sqrt{3}\int_0^2 \left(\tfrac{1}{3}+\tfrac{1}{3}+t+2t^2\right)dt = 26\sqrt{3}/3$$

#3 Find the flux of $F=(x,y,z)$ across the surface S: $2x-2y+z=2$ that is cut by coordinate planes. Use upward normal



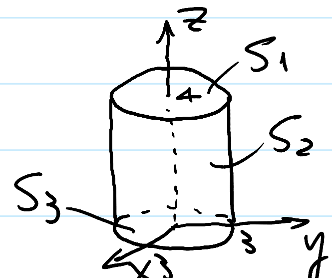
S: $z = 2-2x+2y$, $N = (-g_x, -g_y, 1) = (2,-2,1)$

$F\cdot N = (2x-2y+(2-2x+2y)) = 2$

$$\iint_D F\cdot N\,dx\,dy = 2\iint_D dx\,dy = 2|D| \text{ where } D: \begin{matrix} z=0 \\ y=x-1 \end{matrix} = 2\int_0^1\int_{y=x-1}^{y=0} dx\,\boxed{dy} = 1.$$

#15. $F=(0,0,z)$ $\quad S = S_1 \cup S_2 \cup S_3$

#17. $F=(-y,x,0)$



$$S_{1,3}: \begin{cases} x = s\cos t & 0 \le s \le 3 \\ y = s\sin t & t\big|_0^{2\pi} \ (t\big|_{2\pi}^{0}) \\ z = 4\ (0) \end{cases} \quad N_{1,3} = \begin{vmatrix} i & j & k \\ \cos t & \sin t & 0 \\ -s\sin t & s\cos t & 0 \end{vmatrix} = sk\ (-sk)$$

$$S_2: \begin{cases} x = 3\cos t, & 0 \le t \le 2\pi \\ y = 3\sin t & 0 \le s \le 4, \\ z = s \end{cases} \quad N_2 = T_s \times T_t = \begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ -3\sin t & 3\cos t & 0 \end{vmatrix} = (3\cos t, 3\sin t, 0)$$

#15

On S_1. $F\cdot N = 4s$, $I_1 = \int_0^{2\pi}\int_0^3 4s\,ds\,dt = 36\pi$

On S_2: $F\cdot N = 0$. $I_2 = 0$

On S_3: $F\cdot N = 0$. $I_3 = 0$

$\Rightarrow I = I_1 + I_2 + I_3 = 36\pi$

#17. On S_1 and S_3, $F\cdot N = 0 \Rightarrow I_1 = I_3 = 0$

On S_2, $F\cdot N = (-3\sin t, 3\cos t, 0)\cdot(3\cos t, 3\sin t, 0) = 0$

$\Rightarrow I_2 = 0 \Rightarrow I = I_1 + I_2 + I_3 = 0$.

#21. S: $x^2+y^2+z^2=a^2$, $z \geq 0$. $F = (-y, x, -1)$

Find the flux. S: $z = \sqrt{a^2-x^2-y^2}$.

$N = (-g_x, -g_y, 1) = \left(\frac{x}{\sqrt{a^2-x^2-y^2}}, \frac{y}{\sqrt{a^2-x^2-y^2}}, 1\right)$

$F\cdot N = \frac{-yx+xy}{\sqrt{a^2-x^2-y^2}} - 1 = -1$.

$\text{Flux} = \iint_D F\cdot N\,dx\,dy = \iint_D (-1)\,dx\,dy = -|D|$. D: $x^2+y^2 \leq a^2$, $z=0$ $\Rightarrow |D| = a^2\pi$

$= -a^2\pi$.

#23. S: $X(s,t) = (s\cos t, s\sin t, t)$. $0 \leq s \leq 2$, $0 \leq t \leq 2\pi$, $F = (y, x, z^3)$

Find the flux of F across S.

$N = T_s \times T_t = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos t & \sin t & 0 \\ -s\cos t & s\cos t & 1 \end{vmatrix} = (\sin t, -\cos t, s)$

$F\cdot N = (s\sin t, s\cos t, t^3)\cdot(\sin t, -\cos t, s) = s\sin^2 t - s\cos^2 t + st^3$

$\text{Flux} = \int_0^{2\pi}\int_0^2 (s\sin^2 t - s\cos^2 t + st^3)\,ds\,dt = \int_0^{2\pi}(2\sin^2 t - 2\cos^2 t + 2t^3)\,dt$

$= 2\frac{t^4}{4}\Big|_0^{2\pi} = 8\pi^4$.