1. M311.5.6+

Thursday, March 5, 2020 9:57 AM

Ex: Given $x_1=(1,1,1,1)$, $x_2=(-1,4,4,-1)$, $x_3=(4,-2,2,0)$. Use the G-S process to find an $\perp$-normal basis for span$\{x_1,x_2,x_3\}$.

Sol: We do

1) $u_1 = x_1/\|x_1\| = (\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2})$.

2) $p_1 = \langle x_2, u_1\rangle u_1 = 3u_1 = (\frac{3}{2},\frac{3}{2},\frac{3}{2},\frac{3}{2})$.

$$u_2 = (x_2-p_1)/\|x_2-p_1\| = \frac{(-\frac{5}{2},\frac{5}{2},\frac{5}{2},-\frac{5}{2})}{\frac{5}{2}(1+1+1+1)^{\frac{1}{2}}}$$

$$= (-\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2},-\tfrac{1}{2}).$$

3) $p_3 = \langle x_3, u_1\rangle u_1 + \langle x_3, u_2\rangle u_2$

$$= \tfrac{1}{2}(4-2+2)u_1 + (-2-1+1)u_2 = 2u_1 - 2u_2$$

$$= (1,1,1,1) - (-1,1,1,-1) = (2,0,0,2).$$

$$u_3 = (x_3-p_2)/\|x_3-p_2\| = \frac{(2,-2,2,-2)}{(4+4+4+4)^{\frac{1}{2}}} = (\tfrac{1}{2},-\tfrac{1}{2},\tfrac{1}{2},-\tfrac{1}{2}).$$

the $\perp$-normal basis is $\{u_1,u_2,u_3\}$