Some Numerical Findings

(1) Consider solving the Lane-Emden equation

$$
\begin{cases} \Delta u(x) + u^3(x) = 0, & x \in \Omega, \\ u(x) = 0, & x \in \partial\Omega. \end{cases} \tag{1}
$$

Many solutions have been found. Among them, a ground state (MI=1) focusing mainly on the central corridor was computed. Its existence was proved by others several years late.

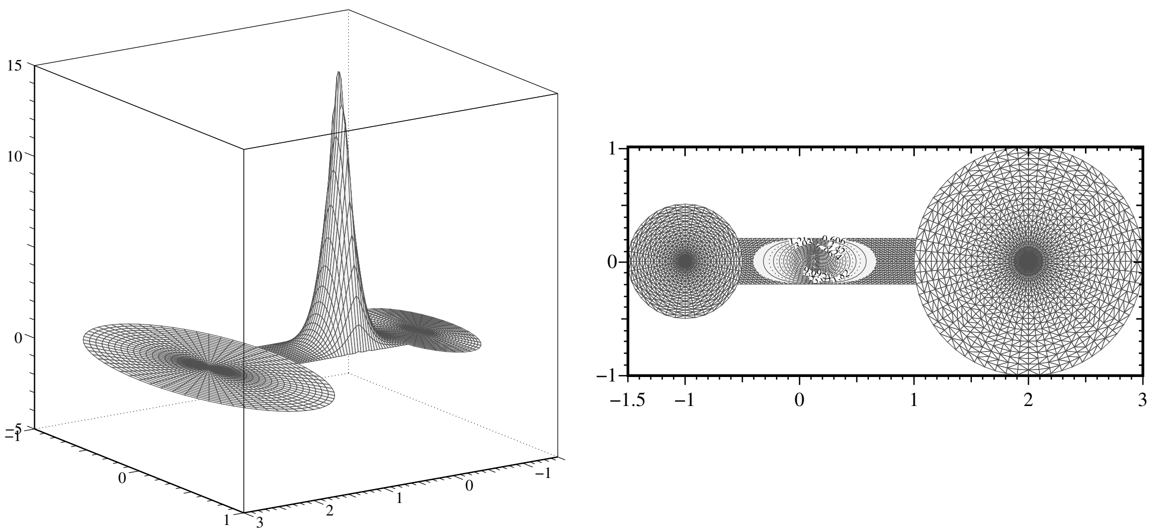


Fig. 1. *The third ground state solution and its contours.*

(2) Consider solving the Henon equation

$$
(2) \qquad \begin{cases} \Delta u(x) + |x|^r u^3 u(x) = 0, & x \in \Omega, \\ u(x) = 0, & x \in \partial\Omega. \end{cases}
$$

Due to explicit dependence of x in the equation, the wellknown Gidas-Ni-Nirenberg theorem about symmetry cannot be applied. Symmetry-breaking phenomenon has been numerically caught when $r > 0.5$. In addition to the radially symmetric positive solution, many radially asymmetric positive solutions are computed. Recently, the relation between the symmetry-breaking phenomenon and the value of r has only been partially verified.

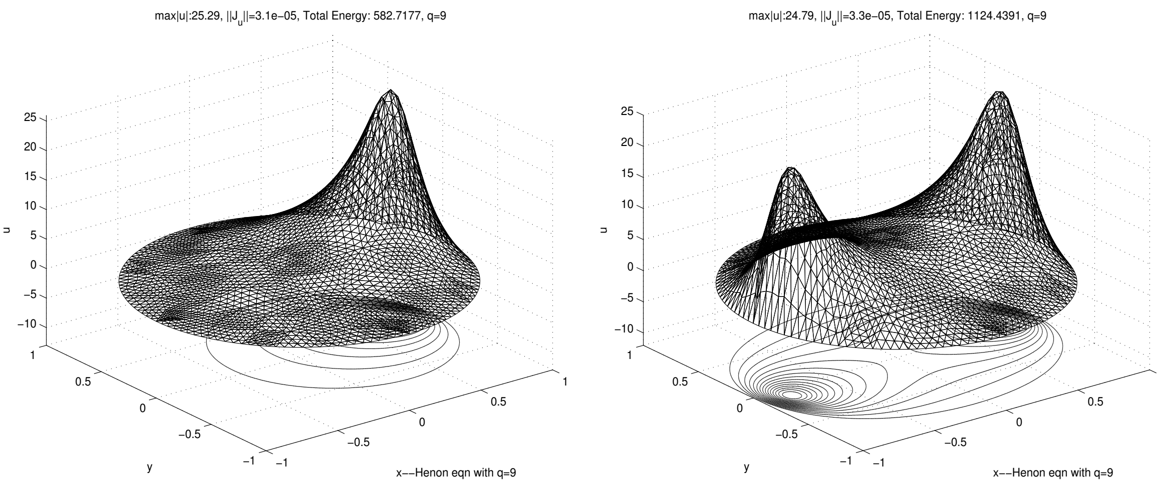


Fig. 2. $r = 9$. *A radially asymmetric ground state and its contours. (left). A 2-symmetric 2-peak solution of* $MI = 2$ *and its contours. (right).*

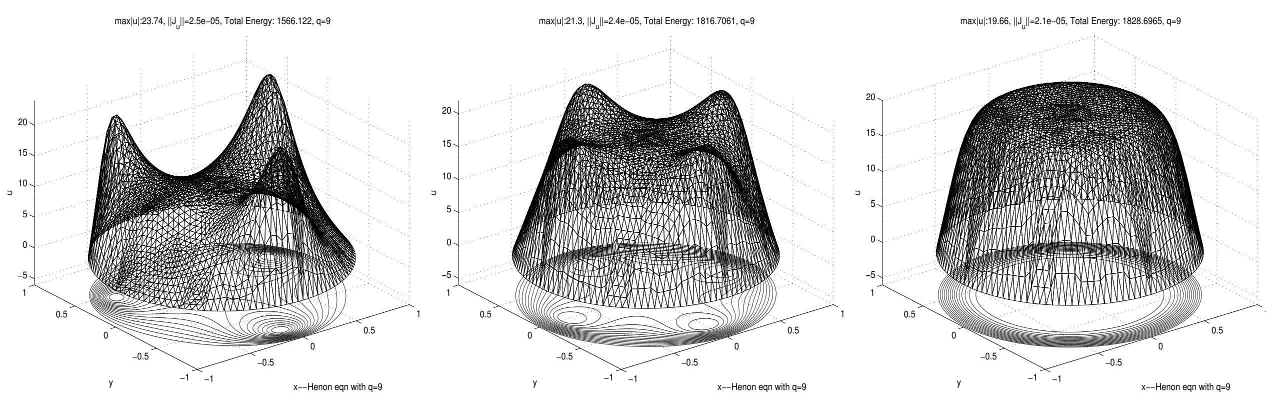


Fig. 3. $r = 9$. *A 3-symmetric 3-peak solution of* $MI = 3$ *and its contours. (left). A 4-symmetric 4-peak solution of* $MI = 4$ *and its contours. (center). A radially symmetric solution of* $MI \geq 5$ *and its contours. (right).*

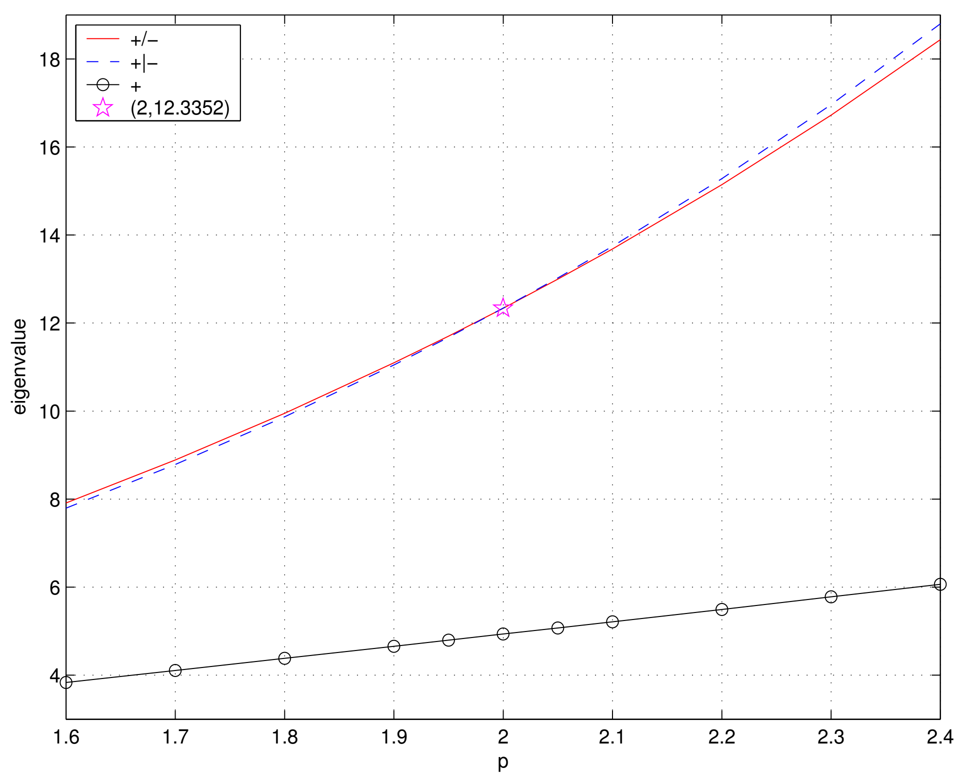
(3) Consider finding the eigenpairs $(\lambda, u) \in R \times (B \setminus \{0\})$ of Δ_p such that

$$-\Delta_p u(x) = \lambda |u(x)|^{p-2} u(x),\ \forall x \in \Omega;\ u|_{\partial\Omega} = 0, \tag{3}$$

where $\Delta_p u(x) = \text{div}(|\nabla u(x)|^{p-2} \nabla u(x))$ is the p-Laplacian operator.

The problems appear in **non-Newtonian fluids** in **rheology**, $p > 1$ is one of the rheological characteristics of the medium, with $p > 2$ are **dilatant** fluids, with $p < 2$ are **pseudoplastics**, with $p = 2$ are **Newtonian fluids**.

Due to nonlinearity involved in derivatives, mathematical analysis and numerical computation are very tough. Many important questions remain open. Thanks to our minimax method in Banach space, we are the first one ever to be able to compute the first a few eigenpairs of Δ_p. We find that when p crosses 2, the patterns of the second and the third eigenpairs of Δ_p switch order. That is why the second eigenvalue of $\Delta = \Delta_2$ is a double eigenvalue. We compute the first three eigenvalues of Δ_p for $1.6 \le p \le 2.4$. Their values have been plotted in the following figure. Many interesting questions can be asked from observing those figures. The first eigenvalues seem to have a linear relation with respect to p.



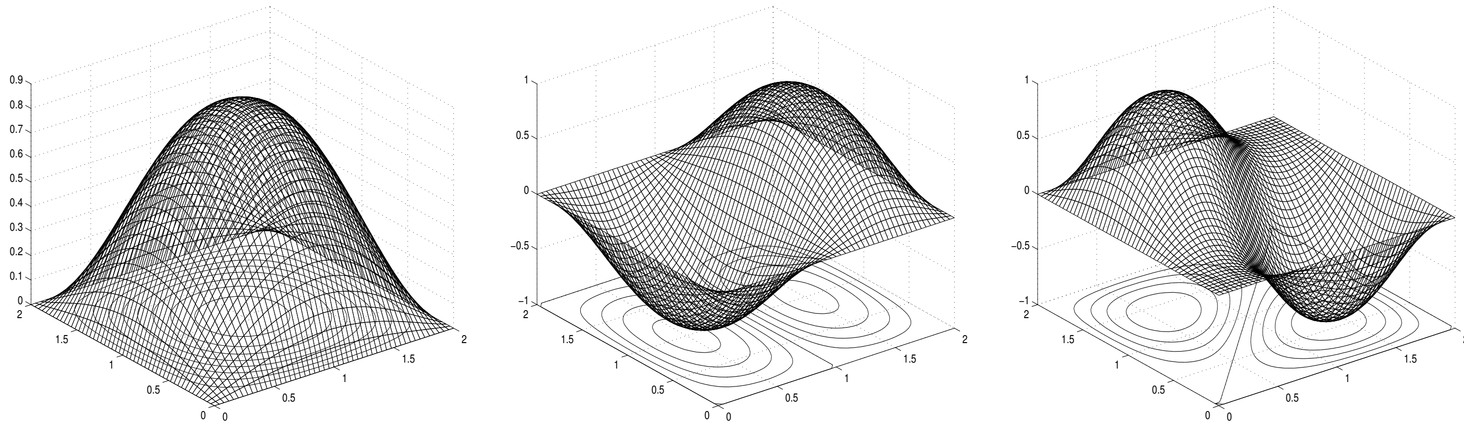


FIG. 4. *The first three eigenfunctions of* $\Delta_p, p = 1.75$.

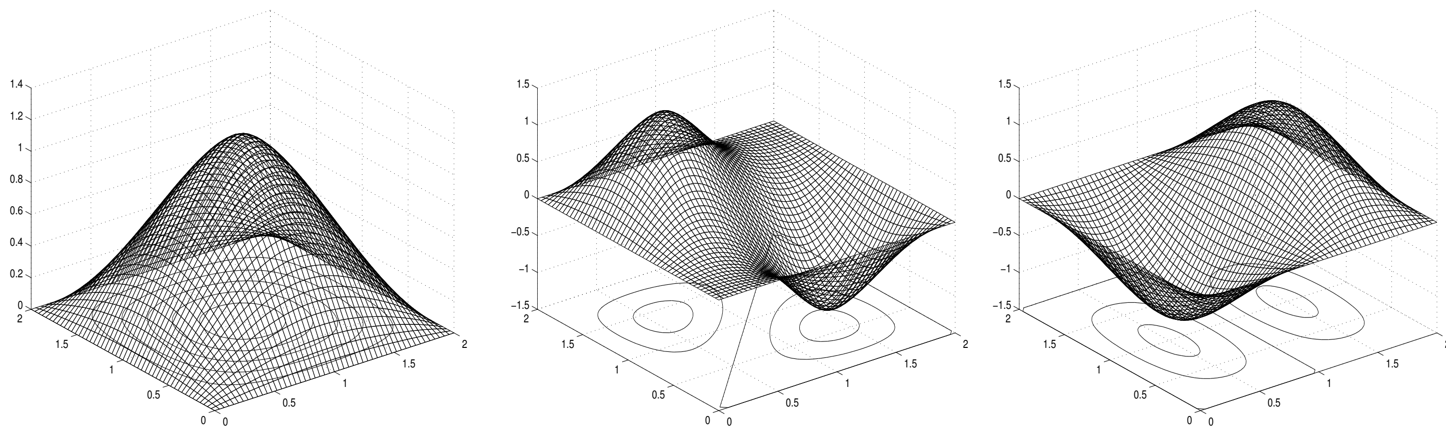


FIG. 5. *The first three eigenfunctions of* $\Delta_p, p = 2.5$.