

SECTION I EXERCISES

1. Use back substitution to solve each of the following systems of equations:

(a) $x_1 - 3x_2 = 2$
 $2x_2 = 6$

(b) $x_1 + x_2 + x_3 = 8$
 $2x_2 + x_3 = 5$
 $3x_3 = 9$

(c) $x_1 + 2x_2 + 2x_3 + x_4 = 5$
 $3x_2 + x_3 - 2x_4 = 1$
 $-x_3 + 2x_4 = -1$
 $4x_4 = 4$

(d) $x_1 + x_2 + x_3 + x_4 + x_5 = 5$
 $2x_2 + x_3 - 2x_4 + x_5 = 1$
 $4x_3 + x_4 - 2x_5 = 1$
 $x_4 - 3x_5 = 0$
 $2x_5 = 2$

2. Write out the coefficient matrix for each of the systems in Exercise 1.

3. In each of the following systems, interpret each equation as a line in the plane. For each system, graph the lines and determine geometrically the number of solutions.

(a) $x_1 + x_2 = 4$
 $x_1 - x_2 = 2$

(b) $x_1 + 2x_2 = 4$
 $-2x_1 - 4x_2 = 4$

(c) $2x_1 - x_2 = 3$
 $-4x_1 + 2x_2 = -6$

(d) $x_1 + x_2 = 1$
 $x_1 - x_2 = 1$
 $-x_1 + 3x_2 = 3$

4. Write an augmented matrix for each of the systems in Exercise 3.

5. Write out the system of equations that corresponds to each of the following augmented matrices:

(a) $\left[\begin{array}{cc|c} 3 & 2 & 8 \\ 1 & 5 & 7 \end{array} \right]$

(b) $\left[\begin{array}{ccc|c} 5 & -2 & 1 & 3 \\ 2 & 3 & -4 & 0 \end{array} \right]$

(c) $\left[\begin{array}{ccc|c} 2 & 1 & 4 & -1 \\ 4 & -2 & 3 & 4 \\ 5 & 2 & 6 & -1 \end{array} \right]$

(d) $\left[\begin{array}{cccc|c} 4 & -3 & 1 & 2 & 4 \\ 3 & 1 & -5 & 6 & 5 \\ 1 & 1 & 2 & 4 & 8 \\ 5 & 1 & 3 & -2 & 7 \end{array} \right]$

6. Solve each of the following systems:

(a) $x_1 - 2x_2 = 5$
 $3x_1 + x_2 = 1$

(b) $2x_1 + x_2 = 8$
 $4x_1 - 3x_2 = 6$

(c) $4x_1 + 3x_2 = 4$
 $\frac{2}{3}x_1 + 4x_2 = 3$

(d) $x_1 + 2x_2 - x_3 = 1$
 $2x_1 - x_2 + x_3 = 3$
 $-x_1 + 2x_2 + 3x_3 = 7$

(e) $2x_1 + x_2 + 3x_3 = 1$
 $4x_1 + 3x_2 + 5x_3 = 1$
 $6x_1 + 5x_2 + 5x_3 = -3$

(f) $3x_1 + 2x_2 + x_3 = 0$
 $-2x_1 + x_2 - x_3 = 2$
 $2x_1 - x_2 + 2x_3 = -1$

(g) $\frac{1}{3}x_1 + \frac{2}{3}x_2 + 2x_3 = -1$
 $x_1 + 2x_2 + \frac{3}{2}x_3 = \frac{3}{2}$
 $\frac{1}{2}x_1 + 2x_2 + \frac{12}{5}x_3 = \frac{1}{10}$

(h) $x_2 + x_3 + x_4 = 0$
 $3x_1 + 3x_3 - 4x_4 = 7$
 $x_1 + x_2 + x_3 + 2x_4 = 6$
 $2x_1 + 3x_2 + x_3 + 3x_4 = 6$