

Matrices and Systems of Equations

(b) If

$$\mathbf{a}_1 = \begin{pmatrix} 1 \\ 1 \\ 3 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{a}_3 = \begin{pmatrix} -2 \\ -1 \\ 1 \\ 3 \end{pmatrix}$$

determine \mathbf{b} .

14. Let A be an $m \times n$ matrix. Explain why the matrix multiplications $A^T A$ and AA^T are possible.
15. A matrix A is said to be *skew symmetric* if $A^T = -A$. Show that if a matrix is skew symmetric, then its diagonal entries must all be 0.

16. In Application 2, suppose that we are searching the database of seven linear algebra books for the search words *elementary*, *matrix*, *algebra*. Form a search vector \mathbf{x} , and then compute a vector \mathbf{y} that represents the results of the search. Explain the significance of the entries of the vector \mathbf{y} .
17. Let A be a 2×2 matrix with $a_{11} \neq 0$ and let $\alpha = a_{21}/a_{11}$. Show that A can be factored into a product of the form

$$\begin{pmatrix} 1 & 0 \\ \alpha & 1 \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ 0 & b \end{pmatrix}$$

What is the value of b ?