SECTION 1 EXERCISES

1. Find the angle between the vectors $\mathbf{v}$ and $\mathbf{w}$ in each of the following:

   (a) $\mathbf{v} = (2, 1, 3)^T$, $\mathbf{w} = (6, 3, 9)^T$

   (b) $\mathbf{v} = (2, -3)^T$, $\mathbf{w} = (3, 2)^T$

   (c) $\mathbf{v} = (4, 1)^T$, $\mathbf{w} = (3, 2)^T$

   (d) $\mathbf{v} = (-2, 3, 1)^T$, $\mathbf{w} = (1, 2, 4)^T$

2. For each pair of vectors in Exercise 1, find the scalar projection of $\mathbf{v}$ onto $\mathbf{w}$. Also, find the vector projection of $\mathbf{v}$ onto $\mathbf{w}$.

3. For each of the following pairs of vectors $\mathbf{x}$ and $\mathbf{y}$, find the vector projection $\mathbf{p}$ of $\mathbf{x}$ onto $\mathbf{y}$ and verify that $\mathbf{p}$ and $\mathbf{x} - \mathbf{p}$ are orthogonal:

   (a) $\mathbf{x} = (3, 4)^T$, $\mathbf{y} = (1, 0)^T$

   (b) $\mathbf{x} = (3, 5)^T$, $\mathbf{y} = (1, 1)^T$

   (c) $\mathbf{x} = (2, 4, 3)^T$, $\mathbf{y} = (1, 1, 1)^T$

   (d) $\mathbf{x} = (2, -5, 4)^T$, $\mathbf{y} = (1, 2, -1)^T$

4. Let $\mathbf{x}$ and $\mathbf{y}$ be linearly independent vectors in $\mathbb{R}^2$. If $\|\mathbf{x}\| = 2$ and $\|\mathbf{y}\| = 3$, what, if anything, can we conclude about the possible values of $|\mathbf{x}^T\mathbf{y}|$?

5. Find the point on the line $y = 2x$ that is closest to the point $(5, 2)$.

6. Find the point on the line $y = 2x + 1$ that is closest to the point $(5, 2)$.

7. Find the distance from the point $(1, 2)$ to the line $4x - 3y = 0$.

8. In each of the following, find the equation of the plane normal to the given vector $\mathbf{N}$ and passing through the point P_0:

   (a) $\mathbf{N} = (2, 4, 3)^T$, $P_0 = (0, 0, 0)$

   (b) $\mathbf{N} = (-3, 6, 2)^T$, $P_0 = (4, 2, -5)$

   (c) $\mathbf{N} = (0, 0, 1)^T$, $P_0 = (3, 2, 4)$

9. Find the equation of the plane that passes through the points

   $$P_1 = (2, 3, 1), \quad P_2 = (5, 4, 3), \quad P_3 = (3, 4, 4)$$

10. Find the distance from the point $(1, 1, 1)$ to the plane $2x + 2y + z = 0$.